VOLUME XLI NUMBER 4

THE ASTROPHYSICAL JOURNAL

An International Review of Spectroscopy and Astronomical Physics

EDITED BY

GEORGE E. HALE
Mount Wilson Solar Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

HENRY G. GALE
Ryerson Physical Laboratory of the University of Chicago

MAY 1915

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

AGENTS
THE CAMBRIDGE UNIVERSITY PRESS, LONDON AND EDINBURGH
KARL W. HIERSEMANN, LEIPZIG
THE MARUZEN-KABUSHIKI-KAISHA, TOKYO, OSAKA, KYOTO

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOL. XLI — CONTENTS FOR MAY 1915 — NO. 4

The Astrophysical Journal is published by the University of Chicago at the University Press during each month except February and August. ¶ The subscription price is $5.00 per year; the price of single copies is 65 cents. Orders for service of less than a half-year will be charged at the single-copy rate. ¶ Postage is prepaid by the publishers on all orders from the United States, Mexico, Cuba, Porto Rico, Panama Canal Zone, Republic of Panama, Hawaiian Islands, Philippine Islands, Guam, Samoan Islands, Shanghai. ¶ Postage is charged extra as follows: For Canada, 30 cents on annual subscriptions (total $5.30), on single copies, 3 cents (total 68 cents); for all other countries in the Postal Union, 62 cents on annual subscriptions (total $5.62), on single copies, 11 cents (total 76 cents). ¶ Remittances should be made payable to The University of Chicago Press, and should be in Chicago or New York exchange, postal or express money order. If local check is used, 10 cents must be added for collection.

The following agents have been appointed and are authorized to quote the prices indicated:

For the British Empire: The Cambridge University Press, Fetter Lane, London, E.C. Yearly subscriptions, including postage, 23*s* each; single copies, including postage, 3*s*. 3*d*. each.

For the Continent of Europe: Karl W. Hiersemann, Königstrasse 29, Leipzig, Germany. Yearly subscriptions, including postage, M. 23.60 each; single copies, including postage, M. 3.20 each.

For Japan and Korea: The Maruzen-Kabushiki-Kaisha, 11 to 16 Nihonbashi Tori Sanchome, Tokyo, Japan. Yearly subscriptions, including postage, Yen 11.25 each; single copies, including postage, Yen 1.50 each.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when they have been lost in transit.

Business correspondence should be addressed to The University of Chicago Press, Chicago, Ill.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Yerkes Observatory, Williams Bay, Wis.

Entered as second-class matter, January 17, 1895, at the Post-office at Chicago, Ill., under the act of Congress of March 3, 1879

PLATE VI

North

E. E. Barnard

NEBULOUS REGION NEAR AND WEST OF 38 OMICRON PERSEI

Center of Plate (1855.0) $\alpha = 3^h 29^m$, $\delta = +31^\circ 15'$

10-inch Bruce telescope of the Yerkes Observatory 1914 November 21, Exposure $6^h 41^m$

Scale { 1 cm = 0°.56 ; 1 in. = 1°.42 }

THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XLI — MAY 1915 — NUMBER 4

A GREAT NEBULOUS REGION NEAR OMICRON PERSEI

By E. E. BARNARD

The study of the large diffused nebulae and the dark regions of the sky is quite impossible with the best visual telescopes. It would seem hopeless to learn much about them with the ordinary means of observation. These diffused nebulosities are very faint to the eye; the contracted field of the telescope allows only a very small portion of such objects to be seen when visible at all. Almost any telescope is too powerful for their successful observation—indeed the more powerful the telescope the more they are diffused and the smaller the portion that can be seen. Our knowledge of the existence of most of these objects would be very small if we depended on the visual telescope for their detection, and the most interesting of them would be entirely unknown.

These remarks apply with almost equal force to the vacant or starless regions of the sky, for though their existence in some cases might be known to the visual telescope (they are due mainly to the absence of very faint stars), their true forms and their relation to the diffused nebulae would, in almost every case, be unknown. It is due to photography, and especially to the work done with the ordinary "portrait lens," that they have become known and that their study has been made possible. The wide field and great

light-grasping power of the portrait lens, coupled with the very long exposures to which the sensitive plate can be subjected, give ideal conditions for the study of the widely diffused nebulosities and the large dark spaces. Attention has been called frequently in this journal and elsewhere to regions of this kind which are of special interest—where apparently an intimate connection exists between the vacancies and the large masses of nebulosity. It has been shown in these papers that there is evidence of the existence of some kind of dark or partly luminous matter between us and the fainter stars which, by obscuring the stars, produces the apparent vacancies, and that the diffused nebulosities, referred to above, are the visible evidence of this matter. Regions of this kind were found in Scorpio, in Ophiuchus, and in Taurus. In *Publications of the Lick Observatory*, **11**, Plate 16, I called attention to a condition like this near the star Omicron Persei in the lower right corner of that photograph. Some of the nebulosity is shown faintly, and it was suggested that a long exposure at that point would perhaps show more of this matter, its presence being indicated by the otherwise unexplained absence of the small stars.

On November 21, 1914, I gave an exposure of 6 hours and 41 minutes on this region with the Bruce 10-inch and 6-inch telescopes. A large, feebly luminous nebulosity with considerable detail in it is shown on these plates. The more obscure parts of this nebula are excessively faint, but the brighter details are well shown. The nebula fits into the vacancy referred to and seems (by obscuring their light) to account for the absence of the small stars. This photograph is reproduced here in Plate VI. It will be noticed, as in other cases to which I have called attention, that in the brighter part of the nebula west of Omicron Persei the background of small stars is continuous. It is only where the nebulosity is very feeble that the stars seem to be more or less missing.

The center of the brightest portion of the nebulosity is in α 3^h29^m, $\delta+31°40'$. In this, about 1° west of Omicron Persei, in α 3^h31^m, $\delta+31°40'$, is a black area nearly a degree in diameter, in which is a bright strip some 20′ long, lying northwest and southeast. In the feebler parts of the nebula, about 40′ south of the dark spot, is a very irregular dark structure.

The great nebulosity, including its feebler portions, extends roughly 7° or 8° east and west and about 3° north and south. Its brighter portion is some 2° in diameter, and consists of cloudlike structures with the dark opening referred to on the south. There are many dark forms in the feebler portions of the nebulosity, especially in the southwestern part. Apparently there is a rift or opening free from nebulosity in α 3^h26^m, $\delta+31\frac{1}{2}°$, through which, seemingly, the background of stars is visible. This includes the stars B.D.+31°616 ($6^m.5$) and +31°619 ($7^m.0$). It is about $1\frac{1}{2}°$ long and $\frac{1}{2}°$ wide.

The nebulosity about B.D.+30°548 (N.G.C. 1333) is roundish and not symmetrical with respect to the star—its center seems to be several minutes to the south. Some faint stars are involved and a thin short strip of nebulosity lies close south of it. The following three nebulous stars are not in any of Dreyer's lists of nebulae. B.D.+30°540 ($8^m.8$) is in a faint and close nebulous atmosphere. B.D.+30°565 ($9^m.1$) is partly surrounded by an irregular, more or less curved nebulous mass. B.D.+31°597 ($7^m.0$) is very closely nebulous. A peculiarity of this nebulous condition is that, on the plate made with the 6-inch lens, its image, compared with that of B.D.+31°599 ($7^m.5$), is relatively much smaller than it is on that with the 10-inch, made at the same time. This is due to the greater scale of the 10-inch.

A nebulosity close south of Omicron Persei is mixed up with a number of small stars, several of which are very close together and appear to be the center of condensation. The principal star of these is B.D.+31°643, of magnitude 8.2 (see *Monthly Notices*, **60**, 261, 1900). It is 7′ south and 3′ following Omicron Persei. This nebulosity extends in a diffused manner beyond Omicron and has a winglike extension south from it for 10′ or 15′. It is very greatly brighter than any portion of the large nebulosity west of Omicron Persei. A small star (not in *B.D.*) 10′ west of Omicron seems to have a very small condensed nebula close west of it. All this region near the stars B.D.+29°565, +30°540 and N.G.C. 1333 is full of feeble nebulosity in which are many dark structures. Some of these are especially noticeable north of B.D. +29°565 and south of +30°540 and also south of N.G.C. 1333.

This great nebula and vacant space near Omicron Persei are but a part of a very remarkable region which includes the Pleiades and is roughly bounded by the following co-ordinates:

$$\alpha\ 3^h4^m \text{ to } \alpha\ 4^h40^m$$
$$\delta +20^\circ \text{ to } \delta +37^\circ$$

In this large region are comprised the present nebulous mass, the "Exterior Nebulosities of the Pleiades" (*Monthly Notices*, **60**, 261, 1900), and the "Nebulous Background in Taurus" (*Astrophysical Journal*, **25**, 218, 1907). To show the relation of these remarkable regions the map on p. 257 has been prepared. Perhaps a word in connection with this chart is required. It roughly shows the region of diffused nebulosity and dark lanes in Taurus, the exterior nebulosities of the Pleiades, and the present nebulous region in Perseus. At the top in the position $\alpha\ 3^h50^m$, $\delta+36^\circ$ is the nebula N.G.C. 1499. (See *Astrophysical Journal*, **2**, 350 and *Publications of the Lick Observatory*, **11**, Plate 16.)

It would need a skilful artist to delineate correctly all the streaky nebulosities that are shown on the original photographs, especially in the region of the Pleiades. I am able to indicate only roughly the general distribution of this matter and the relation of the various parts to each other. For a study of their details and peculiarities in general, I would refer to the *Astrophysical Journal*, **25**, 218, Plates XI and XII, "On a Nebulous Groundwork in the Constellation Taurus," and especially in the case of the Pleiades to *Publications of the Lick Observatory*, **11**, Plate 15, and to *Monthly Notices of the R.A.S.*, **60**, 258, Plates 9 and 10, the latter from a drawing by Mr. E. Calvert, a skilful artist. The title of this last paper is "Exterior Nebulosities of the Pleiades." In the present diagram the nebulosities about the Pleiades should also be shown extending farther to the west. I hope later to investigate that part of the sky more thoroughly and supply the missing nebulosities. I have not attempted to show the nebulosities that are involved in the Pleiades cluster itself.

I get the impression from several of the photographs that some of the luminous streaks to the east of the Pleiades ultimately become the dark lanes shown farther to the east. I hope to be able to confirm this later.

Map of the great Nebulous Regions of Taurus and Perseus

The lower eastern part of Plate VI contains some of the masses which belong to the "exterior nebulosities" of the Pleiades. One of these, a roundish spot, is in α 3^h38^m, $\delta+30°0'$. South of this and near the lower edge of the plate are several irregular masses, the brightest of which is in α 3^h35^m, $\delta+28°50'$. It extends in an irregular manner for several degrees east and west. A fainter patch lies in α 3^h32^m, δ $+29\frac{1}{2}°$.

In connection with the above-mentioned objects several other regions near, which are shown on my various photographs, strongly suggest the presence of similar masses of faint nebulosity or obscuring matter. One of these is a semi-vacant region about 9° long and 3° wide, lying northwest by southeast. Its center is in α 4^h16^m, $\delta+37°$. The stars, which are very rich on the southwest side, abruptly cease and continue again on the northeast side, but their limit is not so abrupt. One gets the impression that this partly vacant region is nebulous, but none of the nebulosity is bright enough to show on the plates I have obtained. The nebula N.G.C. 1579 lies on the south side, near the east end of this region. In another less striking region, in an irregular semi-vacant spot about a degree in diameter, is an elongated dark spot 26′ long by 12′ wide, extending nearly north and south in the position α 4^h21^m, δ $+46°21'$. This small spot occurs in a space free from any stars and suggests the presence of some kind of medium different from the stars.

All the positions in this paper are for the epoch 1855.0.

Following are the *B.D.* positions for 1855.0 of the stars mentioned in this paper.

+31°597 (7.0)	α $3^h16^m27^s.9$	δ +31°12′.8
+30 540 (8.8)	3 16 56.8	+31 25.2
+31 599 (7.5)	3 17 34.6	+31 18.7
+29 565 (9.1)	3 19 29.8	+29 16.5
+30 548 (neb.)	3 20 23.8	+30 53.1*
+31 616 (6.5)	3 25 59.8	+31 31.7
+31 619 (7.0)	3 26 38.8	+31 11.5
+31 642 (3.8)	3 35 16.2	+31 49.6†
+31 643 (8.2)	3 35 27.6	+31 42.2

* N.G.C. 1333. † Omicron Persei.

YERKES OBSERVATORY
WILLIAMS BAY, WIS.
March 29, 1915

A COMPARISON OF THE HARVARD AND MOUNT WILSON SCALES OF PHOTOGRAPHIC MAGNITUDE[1]

By FREDERICK H. SEARES

I. INTRODUCTION

For several years the question of standard photographic magnitudes has received much attention at the Harvard Observatory. Announcements of results have been made from time to time,[2] and recently the details and the final magnitudes derived from a long and painstaking investigation, which has been in the hands of Miss Leavitt, have come from the press.[3] In the meantime, the desirability of applying the 60-inch reflector of the Solar Observatory to problems in photographic photometry led to the formulation of methods of observation and reduction adapted for use with that instrument. In order that there might be some control over the results, the first observations were of the stars of the Polar Sequence, which had already been selected by Professor Pickering. Little by little the investigation broadened, and eventually it developed into an independent determination of the photographic and photo-visual scales, not only of the Polar Sequence, but of numerous other stars near the Pole. This investigation is now finished and has been summarized in a previous paper.[4]

A comparison of the Harvard and Mount Wilson results[5] reveals a very satisfactory agreement between the tenth and the fifteenth magnitudes so far as parallelism of the scales is concerned. For both the brighter and the fainter stars there are differences, which, although not greater than might have been anticipated, are sufficient

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 98.

[2] *Harvard Circulars*, Nos. 125, 150, 160, 170 (1907, 1909, 1910, 1912).

[3] *Harvard Annals*, **71**, No. 3, 1914.

[4] *Mt. Wilson Contr.*, No. 97; *Astrophysical Journal*, **41**, 206, 1915.

[5] *Ibid.* The Harvard results referred to are those of *H.C.*, No. 170, or of the twelfth column of Table LXVII of Miss Leavitt's memoir. They define the scale derived from the photographs measured and reduced at the Harvard Observatory.

to introduce an uncomfortable degree of uncertainty into the statistical investigations for which precise magnitudes are required. To avoid this difficulty Miss Leavitt has combined the magnitudes of *H.C.*, No. 170, with preliminary results from Mount Wilson,[1] and with various other determinations covering a limited portion of the scale, thus forming a system of somewhat greater weight than that based upon the Harvard data alone. But this procedure can be regarded only as a temporary expedient, for the outstanding differences are so large as to indicate serious systematic errors, and until these can be traced to their origin and eliminated no sense of security can be attained.

Now that the details of Miss Leavitt's reduction are in print, it is possible to test various hypotheses which might account for the differences between Mount Wilson and the scale of *H.C.*, No. 170; and it is the purpose of this paper to examine critically the data and the results which she has given. A circumstance of great assistance is the fact that we may certainly regard the Harvard scale between the tenth and the fifteenth magnitudes as substantially correct in slope. Not only is the parallelism with Mount Wilson close, but the agreement with the scales of Chapman and Melotte[2] and of Dziewulski[3] is equally good. It is therefore possible to apply to the discussion methods which could not be used for the initial reduction of the data. For example, with the magnitudes between ten and fifteen as an adopted scale, we can in a number of cases derive results for the brighter stars without making any assumption as to the values of the reduction-constants, since these are determined directly from the data, thus eliminating a considerable portion

[1] *Mt. Wilson Contr.*, No. 70; *Astrophysical Journal*, **38**, 241, 1913. In combining the results of *H.C.*, No. 170, with those of Mount Wilson an arbitrary change has been made in the latter between the ninth and tenth magnitudes. Miss Leavitt has interpreted as a break in the scale what appears to be only a consequence of the accidental errors affecting the preliminary magnitudes. As a result, the Mount Wilson determination of the magnitudes of stars fainter than ten, relatively to the international zero-point defined by those of the sixth magnitude, is disregarded, and Miss Leavitt's modified scale below this limit is substantially that of *H.C.*, No. 170.

[2] *Monthly Notices*, **74**, 40, 1913. These results are not independent of the Harvard scale, as they were obtained with the aid of Harvard standard magnitudes. An independent test by means of gratings gave closely accordant results, however.

[3] *Astronomische Nachrichten*, **198**, 65, 1914.

of the errors affecting the individual plates. And by starting from a series of known magnitudes, we can in other cases, namely, those involving exceptionally large values of the constants, avoid certain difficulties which are inherent in the usual process of reduction.

It is thus possible to re-reduce by independent methods a part of the data upon which the Harvard magnitudes are based. An account of this re-reduction will be found in the following pages. The least satisfactory in its results is that part of the present discussion relating to the stars brighter than the tenth magnitude. The deviation for the faint stars, on the other hand, is easily explained. Since the Harvard scale in this region depends exclusively upon Mount Wilson plates which have also been used in determining the Mount Wilson magnitudes, the divergence must be due to differences in measurement and reduction. Such is found to be the case, and the exact point at which they enter is easily specified.

Before proceeding to the detailed discussion of these two divergent regions of the scale, the effect of color upon the Harvard results must be briefly considered.

II. COLOR-CORRECTIONS AND THE ADOPTED SCALE

The Harvard results are based upon 20 groups of plates for each of which magnitudes on an absolute scale were derived by the methods and instruments listed in Table I. These constitute Section A of the results. Four other sections, namely, B, C, D, E, include results from plates which were reduced to the scale of Section A. Section A therefore determines the scale, while the remaining sections afford material for the reduction of the accidental errors affecting individual stars. The mean of the five sections[1] is the scale of *H.C.*, No. 170. A zero-point correction of +0.08 magnitude was subsequently applied and the results were combined with those of other observers to form the final scale of *H.A.*, **71**. It is, however, the mean result from Sections A to E of Miss Leavitt's investigation plus the zero-point correction of 0.08 magnitude, in which we are at present interested. For convenience

[1] Excepting small differences for a few of the faint stars.

this series of magnitudes will be referred to as the Harvard Scale (HS). Hence,

$$\text{HS} = \text{System A to E} + 0.08 = \text{System } \textit{H.C.}, \text{ No. } 170 + 0.08$$

As stated in *Mt. Wilson Contr.*, No. 97, this scale is not altogether homogeneous with respect to the influence of color. For all cases of known color Miss Leavitt has applied corrections which

TABLE I

HARVARD OBSERVATIONS—SECTION A

Group	Mag. Range	Method	Instrument	Δm	Aperture
					mm
1......	2.7– 6.7	Different foci	11-in. Draper	....	....
2......	2.7– 8.7	Iceland spar	8-in. Draper	2.50	....
3......	4.5– 8.9	Pleiades comp.	Various	....	....
4......	6.4–10.6	Praesepe comp.	1-in. Cooke	....	....
5......	2.7–11.3	Screen B2	1-in. Cooke	2.50	....
6......	8.9–15.5	Screen C	16-in. Metcalf	1.55	....
7......	8.9–13.7	Screens C, D, E, B1	16-in. Metcalf	1.15–2.50	....
8......	8.9–13.3	Whole and half ap.	11-in. Draper	0.75	....
9–12...	2.7–14.5	Prismatic comp.	8-in. Draper	5.00	211–25, 20
13......	2.7–14.3	Pleiades comp.	Various	5.00	211–25, 20
14......	2.7– 7.7	Circ. diaphragm	1-in. Cooke	1.50	8, 4
15......	2.7–11.9	Circ. diaphragm	1-in. Cooke	2.28	26.5, 9.3
16......	8.9–14.3	Circ. diaphragm	11-in. Draper	2.00	280, 112
17......	8.9–15.9	Circ. diaphragm	11-in. Draper	2.00	280, 112
18......	16.0–19.0	Circ. diaphragm	60-in. Refl.	4.81	1511–584, 152
19......	10.5–18.9	Circ. diaphragm	60-in. Refl.	2.97	1511–584, 356
20......	10.5–19.1	Circ. diaphragm	60-in. Refl.	2.97	1511–584, 356

reduce the results to the system of the AC (Cooke 1-inch) and I (Draper 8-inch) instruments. But most of the faint stars, whose colors were then unknown, are red or reddish and also require appreciable corrections. The completion of the reduction to a homogeneous color-system therefore first requires our attention. Although a tedious operation, this is easily accomplished with the color-indices now available.

The color-equations of the various instruments were found by comparing the Mount Wilson color-indices[1] with the differences on pp. 196–198 of Miss Leavitt's memoir.[2] Certain equations control-

[1] *Mt. Wilson Contr.*, No. 97; *Astrophysical Journal*, **41**, 206, 1915.

[2] *Op. cit.* For brevity this will be referred to by the letter L followed by the page number.

ling the consistency of the results were also obtained by comparing the Mount Wilson magnitudes with the various groups of Sections A and B in L, 186–188, 195. To avoid the influence of differences of scale for the bright and the faint stars, only those stars between magnitudes ten and fifteen (Nos. 12 to 14*s*, L, 186–188) were used in the comparisons with Mount Wilson. Disregarding constant terms, the results are as shown in Table II.

TABLE II

COLOR EQUATIONS

Equation	Source
MW − MC = +0.33 C	Section A Group 6, 7
MW − C = +0.08 C	" " 17
MW − I = +0.06 C	" " 9–12
MW − I = +0.06 C	" B " 23–30
I − AI = −0.13 C	" B compared with Section C, Gr. 31
I − ACe = +0.10 C	" " " " " " " 32, 34
I − L = +0.14 C	" " " " " " " 37–39
I − MC = +0.24 C	" " " " " " " 48–53
I − ACe red. = −0.07 C	" " " " " D " 62
I − L = +0.28 C	" " " " " " " 64
I − MC = +0.23 C	" " " " " " " 66
I − ACe = −0.14 C	" A and C revised with E " 68

The symbols in the left members of the equations refer to the instruments listed in L, 55. C in the right members is the Mount Wilson color-index. Beginning with the fifth equation, I includes the AC (later 1-inch Cooke) instrument as well as the 8-inch Draper and signifies the standard system chosen by Miss Leavitt. Owing to a lack of data she was unable to make a direct comparison of the color-systems of these two telescopes; but the differences used in deriving the foregoing equations for L (4-inch Cooke), which include AC as well as I results, show that they afford a really homogeneous system.

Although the coefficient for the second L comparison is twice that of the first, the evidence seems unmistakable; the difference shows clearly in both the AC and the I results. Again, the two ACe comparisons give discordant coefficients; but there seems here also to be a real difference, although the first comparison is not very reliable. On the other hand, the large MC (16-inch Metcalf) correction is constant and well determined. Results from the other

instruments appearing in Miss Leavitt's discussion agree sensibly with the standard system.

The next operation was the revision of the mean magnitudes for each of the five sections. Beginning with Section A the results for each group were corrected with the aid of the foregoing equations. The mean magnitudes for individual stars were then formed exactly as described in L, 185, thus giving results which define the homogeneous scale, just as the original results for this section determined the HS. The remaining sections were then similarly treated, and finally each received a correction to reduce it to the revised scale of Section A, a procedure necessitated by the fact that the color-corrections modify the results for the different sections by different amounts. This reduction was made graphically, and the results for all sections were then collected in a table similar to that in L, 207–209. The weighted means were then found for each star in the manner followed by Miss Leavitt (L, 206) and corrected by +0.08 mag. for zero-point, thus giving what may be called the Harvard Homogeneous Scale (HH). In revising the results care was taken throughout to make allowance for the color-corrections already applied by Miss Leavitt (L, 204).

The agreement of the results from the different sections was very good before the revision, the average deviation of the mean magnitude for a single section being only 0.022 mag.; the reduction to a homogeneous system has decreased this to 0.019 mag., which is an indication of the remaining accidental errors. The main significance of the results, however, is to be found in the fact that, owing to the systematic dependence of color upon magnitude,[1] the scale itself has been appreciably modified. The amount of this change is indicated by the quantities in the fourth column of Table III. The application of these corrections to the values of HS gives the Harvard Homogeneous magnitudes in the HH column.

The comparisons of Mount Wilson with the original results for Sections A and B revealed an appreciable color-equation between the Mount Wilson and AC-I systems. This same relation should

[1] *Mt. Wilson Contr.*, No. 81; *Astrophysical Journal*, **39**, 361, 1914.

appear from the differences MW−HH in Table III; and, in fact, we find

$$MW-HH=+0.24+0.06\ C,$$

which is in agreement with the results in Table II. Star 11*s* (MW 15.31) was the faintest object used in deriving this relation, although all the brighter objects excepting 1*s* were included after applying a correction for scale-difference between HH and MW. This correction has the form

$$MW-HH=+0.061\ (HH-10),$$

which holds for objects brighter than 10.

For the final comparison of the Harvard and Mount Wilson results, we must reduce them to the same color system by means of the first of the foregoing equations. The necessary corrections for the reference of HH to the Mount Wilson system are in the column headed HH_w-HH, and their subtraction from MW−HH shows the outstanding differences of scale between Mount Wilson and Harvard. These appear under the heading $MW-HH_w$ in Table IX of *Mount Wilson Contr.*, No. 97; but for the purpose of the present paper it is convenient to have the two scales coincide from the tenth magnitude onward. On this account the constant term of 0.24 in the color equation was also taken into account in forming the differences $MW-HH_w$ in Table III.

By way of recapitulation, the latter differences indicate the relation between the Harvard (HS) and Mount Wilson scales after the former has been subjected to the following operations: (1) reduction to a homogeneous color system; (2) correction for color to reduce to the MW system; (3) correction for zero-point to produce coincidence with MW in the region of parallelism (mags. 10–15). The last column of Table III shows that in the intermediate region the agreement is very satisfactory. Between the ninth and sixteenth magnitudes there are but two differences exceeding 0.10 mag., namely, those from the red stars 4*r* and 6*r*.

Neither of the color-corrections is very large, but together they form an appreciable quantity, which is important because of its differential effect between bright and faint stars. Both enter in a direction such as to decrease the original divergence between HS

TABLE III

CORRECTIONS FOR COLOR AND COMPARISON OF SCALES

Star	MW	HS	HH *minus* HS	HH	CI	MW *minus* HH	HH_W *minus* HH	MW *minus* HH_W
1*s*....	2.54	2.79	....	(2.79)	0.46	(−25)	....	(−49)
1.....	4.39	4.55	0	4.55	0.02	−16	0	−40
2.....	5.30	5.32	−1	5.31	0.02	−1	0	−25
3.....	5.83	5.82	0	5.82	0.27	+1	+2	−25
4.....	5.91	5.99	0	5.99	0.07	−8	0	−32
2*s*....	6.45	6.46	0	6.46	0.12	−1	+1	−26
5.....	6.45	6.47	0	6.47	−0.02	−2	0	−26
3*s*....	6.64	6.62	+1	6.63	0.29	+1	+2	−25
1*r*....	6.61	6.77	+1	6.78	1.52	−17	+9	−50
6.....	7.11	7.06	+1	7.07	0.06	+4	0	−20
7.....	7.41	7.26	0	7.26	−0.11	+15	−1	−8
2*r*....	7.94	7.82	−1	7.81	1.59	+13	+10	−21
8.....	8.32	8.18	+2	8.20	0.19	+12	+1	−13
3*r*....	8.96	8.70	+1	8.71	1.40	+25	+8	−7
9.....	8.88	8.78	+1	8.79	0.07	+9	0	−15
10.....	9.12	8.97	0	8.97	0.05	+15	0	−9
4*r*....	9.22	9.05	0	9.05	0.96	+17	+6	−13
11.....	9.73	9.50	+2	9.52	0.20	+21	+1	−4
12.....	10.09	9.81	+2	9.83	0.29	+26	+2	0
5*r*....	10.13	9.86	+2	9.88	1.48	+25	+9	−8
4*s*....	10.25	10.07	−1	10.06	0.42	+19	+2	−7
13.....	10.53	10.25	+1	10.26	0.16	+27	+1	+2
6*r*....	10.46	10.25	+2	10.27	1.25	+19	+8	−13
14.....	10.98	10.60	+2	10.62	0.44	+36	+3	+9
7*r*....	10.94	10.65	+1	10.66	1.04	+28	+6	−2
5*s*....	11.09	10.76	+1	10.77	1.03	+32	+6	+2
15.....	11.22	11.02	+1	11.03	0.33	+19	+2	−7
6*s*....	11.38	11.05	+3	11.08	0.65	+30	+4	+2
8*r*....	11.44	11.14	0	11.14	1.00	+30	+6	0
16.....	11.62	11.34	+1	11.35	0.39	+27	+2	+1
17.....	11.87	11.57	+4	11.61	0.59	+26	+4	−2
9*r*....	....	11.64	+2	11.66	1.20	....	+7	....
18.....	12.27	12.00	+3	12.03	0.38	+24	+2	−2
10*r*....	....	12.29	+3	12.32	0.72	....	+4	....
7*s*....	12.61	12.39	+3	12.42	0.51	+19	+3	−8
19.....	12.69	12.36	+3	12.39	0.41	+30	+2	+4
20.....	13.02	12.67	+3	12.70	0.51	+32	+3	+5
21.....	13.33	12.90	+9	12.99	0.85	+34	+5	+5
11*r*....	13.22	12.91	−1	12.90	1.16	+32	+7	+1
22.....	13.44	13.13	+5	13.18	0.63	+26	+4	−2
23.....	13.60	13.28	+4	13.32	0.50	+28	+3	+1
12*r*....	13.84	13.42	+3	13.45	1.38	+39	+8	+7
24.....	13.93	13.58	+3	13.61	0.60	+32	+4	+4
25.....	14.08	13.81	+2	13.83	0.48	+25	+3	−2
8*s*....	14.49	14.14	+4	14.18	0.71	+31	+4	+3
26.....	14.64	14.27	+4	14.31	0.92	+33	+6	+3
9*s*....	14.75	14.41	+4	14.45	0.90	+30	+5	+1
27.....	14.91	14.63	+3	14.66	0.58	+25	+3	−2
10*s*....	15.29	14.96	+3	14.99	0.80	+30	+5	+1
11*s*....	15.31	14.99	+4	15.03	0.96	+28	+6	−2
28.....	15.27	15.05	+3	15.08	0.73	+19	+4	−9

TABLE III—*Continued*

Star	MW	HS	HH *minus* HS	HH	CI	MW *minus* HH	HH_W *minus* HH	MW *minus* HH_W
12s....	15.33	15.05	+3	15.08	0.64	+25	+ 4	− 3
13s....	15.54	15.17	+4	15.21	1.00	+33	+ 6	+ 3
29.....	15.82	15.60	+2	15.62	0.61	+20	+ 4	− 8
14s....	15.99	15.69	+3	15.72	0.92	+27	+ 6	− 3
30.....	16.18	15.98	+2	16.00	0.74	+18	+ 4	−10
31.....	16.41	16.16	+2	16.18	0.79	+23	+ 5	− 6
15s....	16.57	16.36	+1	16.37	0.86	+20	+ 5	− 9
32.....	16.76	16.56	+1	16.57	1.18	+19	+ 7	−12
16s....	16.86	16.66	+1	16.67	1.36	+19	+ 8	−13
33.....	17.06	16.90	+2	16.92	1.09	+14	+ 7	−17
17s....	17.19	17.05	+1	17.06	1.30	+13	+ 8	−19
34.....	17.24	17.19	0	17.19	0.95	+ 5	+ 6	−25
35.....	17.63	17.44	+2	17.46	0.69	+17	+ 4	−11
36.....	17.78	17.69	−1	17.68	0.98	+10	+ 6	−20
37.....	18.01	17.88	−1	17.87	1.20	+14	+ 7	−17
18s....	17.94	17.85	−2	17.83	1.03	+11	+ 6	−19
38.....	18.20	18.25	−2	18.23	1.15	− 3	+ 7	−34
19s....	18.16	18.44	−3	18.41	1.21	−25	+ 7	−56
39.....	18.58	18.61	−3	18.58	1.45	0	+ 9	−33
20s....	18.60	18.64	−3	18.61	1.41	− 1	+ 8	−33
21s....	18.66	18.82	−4	18.78	1.33	−12	+ 8	−44
22s....	18.75	18.87	−4	18.83	1.62	− 8	+10	−42
23s....	18.70	19.05	−5	19.00	1.29	−30	+ 8	−62
24s....	18.88	18.96	−8	18.88	1.54	0	+ 9	−33
40.....	18.87	18.97	−4	18.93	1.58	− 6	+ 9	−39
25s....	18.84	19.11	−5	19.06	1.46	−22	+ 9	−55
41.....	19.02	19.22	−5	19.17	1.55	−15	+ 9	−48

and MW for the bright stars. Thus the unexplained difference between the sixth and tenth magnitudes has been reduced from 0.40 mag.[1] to 0.24 mag.

We now turn to the question whether this remaining divergence, as well as that for the faint stars, is capable of further reduction. In accordance with the explanation on p. 260 we adopt for this investigation, as accurately known, the HH magnitudes between ten and fifteen, and proceed first to an extension of this scale in the direction of the brighter stars.

III. THE DIVERGENCE FOR THE BRIGHT STARS

An examination of Table I shows that the groups of observations there listed may be classified as follows: (*a*) Groups 1–4 and 14, including stars brighter than the ninth magnitude (approximately);

[1] *Mt. Wilson Contr.*, No. 97, p. 16; *Astrophysical Journal*, **41**, 221, 1915.

(*b*) Groups 6–8, 16, and 17, including stars fainter than the ninth magnitude; (*c*) Groups 5, 9–13, and 15, which connect the results from (*b*) with those of (*a*).

The range in (*c*) is from about the third to the fourteenth or fifteenth magnitudes. The zero-point of the magnitudes fainter than 9 depends wholly upon the connecting groups in this division. Groups 18–20 include the fainter stars and are not considered here.

Since the Harvard scale, except for accidental errors, depends exclusively upon the observations of Section A, it is necessary to examine in detail the data for the above-mentioned groups. These we now consider in the order specified.

a) Groups 1–4 and 14

The reductions for this division have not been carefully examined as it is not possible to apply here independent methods of reduction; moreover, the data are not always accessible. The agreement for the different groups is good (see residuals L, 187), although individual plates are sometimes rather discordant (L, 153). One would anticipate that accordance of results by such widely different methods as varying foci, Iceland spar, and circular diaphragms would be strong evidence of a high degree of precision in the mean scale. But we also find close agreement among the Göttingen, Yerkes, and Mount Wilson results[1] for this region (4.0–7.5), and yet these three series of magnitudes show a mean divergence of about 6 per cent from the Harvard scale.

It is of interest to note that the same divergence between Harvard and Göttingen appears when the results for the Pleiades are compared. The Harvard magnitudes for the latter region were derived from Iceland spar and prismatic companion-plates (L, 53, 154, 176) and have been used for the reduction of Groups 3 and 13 of the Polar Sequence. Their deviations from the Göttingen results are shown in the fifth column of Table IV. Inasmuch as the Göttingen series[2] does not include fainter stars, I have extended the scale to the ninth magnitude with the aid of Schwarzschild's Vienna observations.[3]

[1] *Mt. Wilson Contr.*, No. 97.

[2] *Aktinometrie*, B, p. 14, 1912.

[3] "Beiträge zur photographischen Photometrie der Gestirne," *Publ. der von Kuffner'schen Sternwarte*, **5**, 62, 1900.

For further comparison, Hertzsprung's magnitudes[1] of the brighter Pleiades stars (diminished by 0.26 mag.) are also included in Table IV. Although derived by different methods, the Göttingen and Hertzsprung series are in close agreement, and hence the latter also shows the characteristic deviation from Harvard.

TABLE IV

COMPARISON OF SCALES FOR THE PLEIADES

Harvard No.	Bessel No.	Gött.	Hertz.	Gött.–H.	Hertz.–H.
1.........	*η*	2.87	2.85	–14	–16
2.........	*f*	3.57	3.59	– 9	– 7
3.........	*b*	3.59	3.67	– 8	0
4.........	*c*	3.82	3.85	–12	– 9
5.........	*d*	4.21	4.13	– 7	–15
6.........	*e*	4.22	4.20	– 7	– 9
7.........	*h*	4.94	....	+ 4	....
8.........	*g*	5.37	5.38	+ 5	+ 6
9.........	*k*	5.69	5.70	– 3	– 2
10.........	*l*	6.35	6.34	+ 4	+ 3
11.........	24	6.84	6.87	+ 3	+ 6
12.........	29	7.02	7.01	+ 2	+ 1
13.........	4	7.25	7.28	– 1	+ 2
14.........	10	7.43	7.38	+ 6	+ 1
15.........	39	7.57	7.52	+16	+11
16.........	37	7.70	7.55	+14	– 1
17.........	33	8.15	8.14	+16	+15
18.........	....	....	8.67	....	+28
19.........	21	8.94	8.82	+25	+13
20.........	....	....	8.86	....	+ 6
21.........	2	9.00	8.95	+14	+ 9
23.........	36	9.26	9.10	+10	– 6

Some caution must be exercised, however, in judging of scale differences in cases like the present, for there is a progressive change in color with increasing magnitude, and hence a possibility of confusing a color-equation with differences in scale. An examination of the circumstances seems to exclude such an explanation, however.

b) Groups 6–8, 16, and 17

As the scales established by these groups usually begin at the ninth magnitude, they cover but one magnitude of the divergent region which includes the brighter stars. The calculation for the

[1] *Astronomische Nachrichten*, **186**, 181, 1910.

few stars falling within this interval is easily controlled with the aid of the adopted magnitudes in the region 10–15.

The plates show at least two exposures—full aperture (*a*) and reduced aperture (*b*). The Harvard scale readings for (*a*) and (*b*) were plotted against the adopted magnitudes as ordinates, beginning in each case with Star 13 (10.26). The two curves thus defined should be parallel, and the difference in corresponding ordinates should equal the reduction constant Δm. Any deviation from parallelism, or any lack of agreement with the adopted constant Δm, indicates an abnormality which must affect the scale derived from the plate in question. Non-parallelism of the curves signifies differences in gradation for the two exposures. Lack of equality between the ordinate-difference and Δm means that the effective reduction-constant differs from the adopted value. This may be caused by changes in transparency or in the sensitiveness of the plate during the exposures. With the method of reduction here used this is of no consequence, for as long as the curves are parallel, or nearly so, it is only necessary to displace the *a*-curve vertically until it coincides with the *b*-curve. This automatically extends the *b*-curve into the region of smaller scale-readings and gives a curve from which the magnitudes of the brighter stars of the *b*-exposure may be interpolated. The results are independent of changes in transparency, of the phenomenon affecting first and last exposures,[1] and of errors in the adopted reduction-constants.

By an extension of this process it is also possible to make use of scale-readings upon the images of the bright stars shown by the *a*-exposure, when such are present. This tends to diminish the accidental errors, but does not otherwise contribute toward a knowledge of the scale. A complete reduction of all the plates was made by both processes before the necessity of a detailed examination of the influence of color became apparent, although only the results by the simpler process first outlined have been used. A third reduction was subsequently made for Groups 6 and 7, which relate to the Metcalf 16-inch and are most likely to be influenced by color. But inasmuch as the first two reductions had been based upon white stars alone, the final reduction of Groups 6 and 7, in which

[1] *Mt. Wilson Contr.*, No. 64, p. 7; *Astrophysical Journal*, **36**, 374, 1912.

the influence of color was taken into account, made no appreciable difference.

Nearly all of the plates in the five groups of this division appear to be of very satisfactory quality. The magnitude-curves deviate from parallelism by only a few hundredths of a magnitude, and in all cases the calculated reduction-constant is nearly equal to the value adopted by Miss Leavitt.

The results for each plate are shown in Table V in the form of corrections to the HH magnitudes of Table III. The last two lines exhibit the agreement between the observed and the theoretical reduction-constants. The first series of values for Plate 229 depends upon the mean of two reduced-aperture exposures with screens C and D, respectively (L, 159), and has accordingly been given double weight. Plate 186 has also received double weight because of exceptional quality (L, 63) and duplicate measures. In addition to the four stars listed in Table V, corrections were derived from Plate 248 for four others as follows:

No. 6, −21; No. *2r*, −29; No. 8, −24; No. 9, −24

The mean corrections are characterized by a rather marked persistence of the negative sign. The probable errors, which are expressed in thousandths of a magnitude, suggest that the HH magnitude for Star 10, and perhaps that for No. 12, actually require small corrections.

c) *The Connecting Groups 5, 9–13, and 15*

We now consider the important groups connecting the bright stars with those fainter than the tenth magnitude. With the exception of Group 13, the data have been independently reduced, partly by methods equivalent to those used by Miss Leavitt, and partly by other processes.

Group 5 (wire screen B2, $\Delta m = 2.50$, on 1-inch Cooke anastigmat, two exposures of 30^{m}).—Since the faintest star registered on the three plates of this group is 11.2, the magnitudes between 10 and 15 are of no assistance in deriving the scale or in testing the value of the reduction-constant. The usual method of reduction[1] has therefore been applied.

[1] *Mt. Wilson Contr.*, No. 80, p. 24; *Astrophysical Journal*, **39**, 330, 1914.

TABLE V

CORRECTIONS TO HH FROM DIVISION (*b*)

Star	Group 6			Group 7					Group 8				Group 16			Group 17		Mean	P.E.	Wt.
	231	232	233	220		240	243	248	166	167	170	171	182	184	185	186	194			
10........	−10	−22	+5	−26	−23	−13	+7	−3	−24	+11	+24	−9	−6	−15	−6	−10	−12	−9	019	19
11........	−2	+24	+3	−15	−6	−5	+14	+14	.....	.....	−6	.....	.....	+9	−5	−2	+3	+1	018	15
12........	−9	+17	0	−27	−8	−1	+2	+8	−3	−18	−5	+15	−5	+2	−6	−11	+9	−4	017	19
4*s*.......	−19	+6	+11	+5	−14	−20	+6	0	.....	.....	−15	.....	.....	.....	.....	.....	+1	−3	026	11
Obs. Δm..	1.60	1.50	1.43	1.50	1.12	1.52	2.40	2.52	0.73	0.72	0.77	0.80	2.00	1.95	1.90	2.10	1.86			
Theor. Δm.	1.55	1.55	1.55	1.56	1.15	1.55	2.50	2.50	0.75	0.75	0.75	0.75	2.00	2.00	2.00	2.00	1.90			

Although there are differences for individual plates, the mean scale for the group is substantially that found by Miss Leavitt, from which it may be inferred that our reduction-processes are equivalent. The zero-point was determined by making the mean magnitudes of the eight or ten stars fainter than magnitude 10 equal to the mean of the corresponding HH magnitudes. For stars brighter than magnitude 10 there is a preponderance of negative corrections (i.e., the HH magnitudes are relatively too faint) and a divergence in the direction of the Mount Wilson results (compare residuals, L, 187).

A point of fundamental importance is the reduction-constant, which was calculated from the measured dimensions of the mesh on the assumption that the absorption in magnitudes for point sources is twice that for luminous surfaces. The screen is of rather fine wire gauze (L, 159): wires 0.0940 mm, spaces 0.1206 mm), and for dimensions such as these the assumption that the constant, Δm, is twice the absorption for luminous surfaces is not entirely justifiable. Du Bois and Rubens have investigated the question[1] and their results indicate that for this case the double of the absorption for surfaces should be increased by 0.05 mag., in order to obtain the constant for point-sources. This correction makes the magnitudes of the bright stars relatively brighter, the change between 6 and 10 being 1.6 times the correction, or 0.08 mag. The final results, which are based upon the revised constant ($\Delta m = 2.55$), appear in the fourth column of Table X. As before, they are expressed as corrections which, applied to the HH magnitudes, will yield the values derived from the re-reduction of the group.

Groups 9–12 (prismatic companions, with the 8-inch Draper telescope).—For Groups 9–11, the exposures, with one exception, were 10^m. For Group 12, they were 60^m, excepting two plates which received 93^m and 120^m, respectively. The reduction-constant calculated from the clear aperture of the prism (cemented to the center of the objective) and the unobstructed area of the objective is 5.10 magnitudes. This requires correction for differential absorption owing to the different thicknesses of glass traversed by the two beams—that directly transmitted and that deflected by the

[1] *Annalen der Physik*, **49**, 593, 1893.

prism. It was not found possible to determine this directly, and in consequence the following procedure was adopted (L, 172):

It was shown above that the center of the objective of the 11-inch Telescope transmits more light than the average for the entire lens by at least 0.1 magn. If we assume a similar effect in both the lenses of which the 8-inch Draper Doublet is composed, we shall have a correction of −0.2 magn. to the computed difference. On the other hand, we may allow a loss of light of 0.1 magn. arising from the reflection and absorption of the small prism. The adopted difference is $5.1-0.2+0.1=5.0$.

The fact that the absorption for the center of the 11-inch Draper objective is less than the average for the whole objective indicates that the main factor involved is not absorption in the ordinary sense, but residual aberrations in the optical system. These, however, are peculiar to each instrument; and the assumption that what applies to the 11-inch Draper also applies, and in the manner indicated, to the 8-inch instrument, seems open to question. One is therefore inclined to suppose that the adopted value of the constant is subject to some uncertainty. The circumstance is unfortunate, as the bulk of the material (23 plates) connecting the fainter stars with the international zero-point is contained in these four groups.

The matter is further complicated by the fact that the reduction of plates involving so large a constant presents special difficulties. These are pointed out by Miss Leavitt, and to meet them in so far as possible she has employed a special process (L, 147).

The uncertainty arising from this source can now be removed by a method similar to that described on p. 270. With the aid of the adopted scale between 10 and 15 we can derive for the brighter stars magnitudes which are free from systematic errors arising from the reduction-process; but in this instance we are obliged to assume the validity of the reduction-constant, for only a few of the brighter stars of known magnitude show prismatic companions.

We plot the scale-readings of the primary exposures against the HH magnitudes which are fainter than 10, and read from the curve thus defined the magnitudes corresponding to the scale-readings of the prismatic companions. The subtraction of five magnitudes from each value gives the required results. Prismatic companion

readings which do not fall within the limits of the curve cannot at once be utilized. Nor can the readings upon bright stars of the primary exposure be used directly; but by plotting these against the corresponding magnitudes found by the process just described, a curve will be defined which should be an extension of that originally drawn. If, however, an erroneous value of Δm has been used, or if there are differences of gradation between the two exposures, a discontinuity will occur, and the curves must be adjusted so that a smooth junction is effected.

The use of the adjusted curves tends to eliminate gradation-differences and reduce the accidental errors of observation, but in the mean of several plates probably does not materially affect the result. They have not been used in the present case, because the majority of the bright stars are beyond the limit of distance from the center of the plate for which the primary images may be used, and, further, because it was desired to bring into view any peculiarities affecting the method of prismatic companions.

The corrections to the HH magnitudes for the individual plates of the four groups are in Tables VI and VII. The preponderance of negative signs is at once evident and indicates a systematic difference between the results of the present reduction and that of Miss Leavitt. The difference appears clearly in the mean residuals for Groups 9–11, although not in those of Group 12 (Table VIII). For individual plates, the difference is not constant, but shows a marked progression—see, for example, Plate 84 of Group 9 and Plate 116 of Group 12—which suggests either differences in gradation between the two series of images or systematic errors of measurement which easily arise when the secondary images are not closely comparable with those of the primary series. A difference in the slope of the HH scale for the bright stars as compared with that in region of magnitudes 10 to 15 would also produce a progressive change in the residuals; but any such progression would be the same for all the plates.

The systematic difference between the mean residuals for Groups 9–11 and Group 12 is rather remarkable. Two circumstances may be noted: first, the average exposure-time for the last group was about six times that for the others, and, second, four of the plates of

Group 12 either presented some abnormality or were difficult to measure (L, 102, remarks). On the other hand, Plates 98 and 103

TABLE VI

RESIDUALS, GROUPS 9 AND 10

Star	Group 9					Group 10				
	84	85	94	95	104	105	106	107	113	118
2s.......	+14	−18	+ 8	+16	+ 8	......	......	......	......	......
5.......	+13	+ 1	+11	+ 6	−23	−34	−27	−27	−17	+ 6
6.......	+15	+ 1	−13	−14	+16	−29	+ 2	−23	−13	+14
7.......	+16	−24	−23	+32	− 4	......	−18	−27	−18	−13
2r.......	−18	−15	− 5	−14	......	−25	+ 9	−26	−13	−13
8.......	−24	−22	−20	−51	− 7	− 5	− 5	− 6	−30	+ 7
3r.......	−27	−11	− 2	−24	......	......	......	......	......	......
9.......	−14	− 9	−22	−10	− 7	+ 8	−12	−48	+ 5	− 3
10.......	−43	− 6	− 9	−41	−16	−18	−19	− 5	+ 9	−28
4r.......	......	......	− 4	−37	− 5	−19	−32	−47	− 7	− 5
11.......	−25	+15	+ 7	− 8	0	......	−26	−19	......	−33
12.......	......	− 5	− 3	−18	......	......	−26	−39	......	......
5r.......	......	......	......	−16	......	......	......	......	......	......
4s.......	......	......	......	−17	......	......	......	......	......	......

TABLE VII

RESIDUALS, GROUPS 11 AND 12

Star	Group 11					Group 12							
	97	98	99	103	109	108	114	121	148	150	115	116	117
2s..	.....	−24	.....	+ 2	.....	.....	.....	+24	.....	.....	.....	.....	....
5...	+10	−21	.....	−21	−42	+23	+ 1	+23	+37	+68	+ 5	−14	+ 3
1r..	.....	.....	.....	.....	.....	.....	−33	.....	.....	.....	.....	.....	....
6...	+ 8	−27	−41	−12	+ 4	+12	0	+25	−11	+20	+18	−21	+ 4
7...	.....	−29	.....	−17	+ 9	−15	− 6	+40	.....	.....	.....	0	....
2r..	+19	−10	−26	−33	+ 6	.....	−21	+17	.....	.....	− 3	− 3	+ 7
8...	+14	−20	− 4	−54	− 5	−29	− 9	+24	+26	+ 9	−22	+11	− 4
3r..	.....	+ 4	.....	−34	.....	−17	.....	+ 4	.....	.....	.....	+ 3	....
9...	+ 2	.....	− 1	−39	+18	−15	−10	+ 5	0	−18	−17	+ 5	− 3
10...	−27	−12	− 9	− 4	+21	−13	+ 2	−14	− 9	+ 1	− 8	− 2	+19
4r..	.....	−34	.....	−25	+13	−32	− 6	− 2	.....	.....	− 6	+13	+32
11...	.....	.....	−13	−35	−12	− 4	−21	+ 4	+25	− 6	− 8	+ 6	+20
12...	.....	.....	.....	.....	.....	−44	+14	− 6	+13	+ 5	−29	− 1	+ 1
5r..	.....	.....	.....	.....	.....	+16	.....	− 1	.....	.....	.....	+11	....
4s..	.....	.....	.....	.....	.....	−39	−12	−22	− 2	−21	− 8	+31	−37

were measured more than the usual number of times and by two different observers (L, 62, 63).

One is tempted to speculate as to the possibility of an influence depending upon exposure-time, for it is conceivable that the image of a prismatic companion might increase in diameter with increasing exposure at a rate different from that of the image produced by the remainder of the objective; but the data are too slender to permit any definite conclusion.

Group 13.—The results from this group were obtained by comparisons of the Pole with the Pleiades, for which a scale was established by the method of prismatic companions. As there are no

TABLE VIII

MEAN RESIDUALS, GROUPS 9-12

Star	HH Mag.	Mean Residuals				No. Values	Wtd. Mean	P.E.	Wt.
		9	10	11	12				
2*s*	6.46	+6	.....	−11	+24	5, 0, 2, 1	+4	038	8
5	6.47	+2	−20	−18	+18	5, 5, 4, 8	−1	035	22
1*r*	6.78	.....	.....	.....	−33	0, 0, 0, 1	−33	.....	1
6	7.07	+1	−10	−14	+6	5, 5, 5, 8	−3	027	23
7	7.26	−1	−19	−12	+5	5, 4, 3, 4	−6	034	16
2*r*	7.81	−13	−14	−9	−1	4, 5, 5, 5	−9	024	19
8	8.20	−25	−8	−14	+1	5, 5, 5, 8	−10	028	23
3*r*	8.71	−16	.....	−15	−3	4, 0, 2, 3	−12	035	9
9	8.79	−12	−10	−5	−7	5, 5, 4, 8	−8	019	22
10	8.97	−23	−12	−6	−3	5, 5, 5, 8	−10	020	23
4*r*	9.05	−15	−22	−15	0	3, 5, 3, 6	−12	035	17
11	9.52	−2	−26	−20	+2	5, 3, 3, 8	−7	026	19
12	9.83	−9	−32	.....	−6	3, 2, 0, 8	−11	037	13
5*r*	9.88	−16	.....	.....	+9	1, 0, 0, 3	+2	046	4
4*s*	10.06	−17	.....	.....	−14	1, 0, 0, 8	−14	041	9
Mean	....	−9	−16	−12	0		−7		

Mount Wilson results for the Pleiades, a direct comparison cannot be made, but the divergence of the Harvard magnitudes for this region from those of Göttingen and Potsdam has already been referred to in connection with Table IV. As the method used for Groups 9–12 and several of the plates included in Group 11 (Nos. 97–99, 103) were also used for Group 13, it is to be inferred that the results found above also apply here.

Group 15.—This includes four plates taken with a circular diaphragm of 9.3 mm placed centrally over the 26.5 mm objective of a Cooke anastigmat. Two exposures of 45^{m} were made on each

plate. The calculated constant, 2.28 magnitudes, was used without correction for differential absorption within the objective. A re-reduction by the normal method, using the foregoing value of the constant, gives substantially the result found by Miss Leavitt. For two of the plates, however, the deviations are very large (see residuals, L, 178).

To test further the agreement with the HH scale, readings for both diaphragm and full aperture exposures were plotted against the HH magnitudes, using all the stars measured. From the two curves thus defined were read the differences in the ordinates for each half-interval of scale-reading. The deviations of these observed values of Δm from the adopted constant are shown in Table IX. The agreement for Plate 54 alone is really satisfactory,

TABLE IX

GROUP 15, OBSERVED Δm − ADOPTED Δm

Scale Reading	Plate 54	Plate 55	Plate 56	Plate 58
3.0	....	−23	....	....
3.5	....	−18	....	....
4.0	+7	−21	+26	....
4.5	+4	−23	+20	....
5.0	+2	−26	+12	+49
5.5	+2	−26	+ 5	+46
6.0	+2	−28	+ 6	+42
6.5	+2	−28	0	+42
7.0	−3	−28	+ 3	+41
7.5	−5	−20	+ 3	+32
8.0	−3	−16	+ 7	+16
8.5	−4	−13	+17	− 8
9.0	−4	0	+22	−13
9.5	−6	+11	+12	− 8
10.0	0	+16	− 3	+ 2

while Plates 55 and 58 are seriously discordant. Although the errors for these two balance each other, they reduce considerably the weight of the mean scale for the group. Changes in the observed reduction-constant of the character of those shown in Table IX are an indication of differences in gradation or of lack of comparability in the two series of images. Their appearance in the present instance is not surprising in view of the long exposures involved. In addition to these irregularities there is the uncertainty in the reduction-constant arising from differential absorption,

so that Group 15 contributes little weight to the determination of the scale.

Summary for plates connecting bright stars with those fainter than the tenth magnitude.—The preceding paragraph completes the discussion of the data which connect the faint stars with those in the vicinity of the sixth or seventh magnitude. As the zero-point for all the fainter objects must be derived through this connection, its importance for the final scale is obvious. In the region covered there is considerable difference between the HH and MW scales, and the data and reductions have been examined for evidence which might afford an explanation of the divergence. The following summarizes the results:

Three methods have been employed—a wire gauze screen, prismatic companions, and a circular diaphragm. In none of these is it clear that the reduction-constant is certainly free from suspicion. That of the screen is derived from measures of the mesh and apparently requires a correction of $+0.05$ magnitude. In the case of the other two methods, differential absorption in the objective is not wholly accounted for; for the prismatic-companion plates a correction of doubtful validity is used, while for the diaphragm plates the effect is disregarded altogether. As for the prismatic companion constant, the error is probably small in spite of the assumptions involved, otherwise the portion of the scale below the tenth magnitude thus established would not agree so well with the HH scale, which in this region must be accepted as substantially correct.

With the revised screen-constant, we find that the scale thus established agrees closely with MW (compare MW−HH, Table III, with corrections in the fourth column of Table X). Re-reducing the prismatic companion plates with the aid of the adopted scale from 10 to 15, we find a systematic deviation from the Harvard Homogeneous Scale in the direction of the Mount Wilson results, but less, in the mean, than the differences between HH and MW. The examination of the diaphragm plates in Group 15 shows that the mean result agrees well with HH; but two of the plates show serious internal inconsistencies. On the whole, the re-reduction of the connecting plates points toward the necessity for some

correction of the HH scale in the region of the brighter stars. The indicated change is in the same direction as that derived by comparison with MW, but somewhat less in amount.

d) *Results of the Re-reduction*

The corrections to the HH magnitudes found in sections *a*), *b*), and *c*) for all of the groups re-reduced excepting No. 15 are brought together in Table X, their relative weights (number of values)

TABLE X

COLLECTED RESULTS OF RE-REDUCTION—CORRECTIONS TO HH MAGNITUDES

Star No.	HH Mag.	RE-REDUCTION *minus* HH				Wt.	MW *minus* HH_W	MW *minus* Re-reduction
		Gr. 6–8, 16, 17	Gr. 5	Gr. 9–12	Wtd. Mean			
1*s*...	(2.79)	....	−53 (3)	....	−53	3	(−49)	(+ 4)
1....	4.55	....	−33 (3)	....	−33	3	−40	− 7
2....	5.31	....	−30 (3)	....	−30	3	−25	+ 5
3....	5.82	....	−20 (3)	....	−20	3	−25	− 5
4....	5.99	....	−20 (3)	....	−20	3	−32	−12
2*s*...	6.46	....	−19 (3)	+ 4 (8)	− 2	11	−26	−24
5....	6.47	....	−22 (3)	− 1 (22)	− 4	25	−26	−22
3*s*...	6.63	....	−15 (3)	....	−15	3	−25	−10
1*r*...	6.78	....	−20 (3)	−33 (1)	−23	4	−50	−27
6....	7.07	−21 (1)	−14 (3)	− 3 (23)	− 5	27	−20	−15
7....	7.26	....	− 3 (3)	− 6 (16)	− 6	19	− 8	− 2
2*r*...	7.81	−29 (1)	−13 (3)	− 9 (19)	−10	23	−21	−11
8....	8.20	−24 (1)	+ 2 (3)	−10 (23)	− 9	27	−13	− 4
3*r*...	8.71	....	− 6 (3)	−12 (9)	−10	12	− 7	+ 3
9....	8.79	−24 (1)	−17 (3)	− 8 (22)	−10	26	−15	− 5
10....	8.97	− 9 (19)	−13 (3)	−10 (23)	−10	45	− 9	+ 1
4*r*...	9.05	....	−12 (3)	−12 (17)	−12	20	−13	− 1
11....	9.52	+ 1 (15)	−14 (3)	− 7 (19)	− 4	37	− 4	0
12....	9.83	− 4 (19)	+ 3 (3)	−11 (13)	− 6	35	0	+ 6
5*r*...	9.88	....	− 8 (3)	+ 2 (4)	− 2	7	− 8	− 6
4*s*...	10.06	− 3 (11)	− 8 (3)	−14 (9)	− 8	23	− 7	+ 1

being indicated by the quantities in parentheses. Had Group 15 been included, several of the mean corrections in the sixth column would have been decreased one- or two-hundredths of a magnitude; but this change probably would have been offset by one in the opposite direction had it been possible to reduce Group 13 by the foregoing methods. In view of the uncertainties affecting Group 15 it is likely that the results are more reliable as they stand.

The agreement from the seventh magnitude downward of the third, fourth, and fifth columns seems to show pretty clearly the necessity for corrections of the order indicated; but the relation of the fainter HH magnitudes to the international zero-point is unfortunately less definitely established. The results from the prismatic-companion plates (Groups 9–12), which are the most numerous of all the connecting plates, become relatively uncertain just at the critical point, for here the magnitudes are at the end of the scale derived from these groups. The only groups in Miss Leavitt's data which satisfactorily cover the questionable region are Nos. 3, 5, 13, and 15. The last appears to be unreliable for reasons given above. Nos. 3 and 13 involve comparisons with the Pleiades, and, although accordant with each other, do not agree with the results from Group 5.

The corrections in the sixth column, with the exception of that for Star 12, are all such as to decrease the outstanding differences between MW and HH. With the corrections included, the comparison with MW is as shown in the last column of Table X. The MW scale is therefore in agreement with the results of the re-reduction of the Harvard data from about the eighth magnitude downward. Above the seventh magnitude the differences are uncertain, either because of the small number of observations included in the re-reduction or because the calculated magnitudes are at the extremity of the scale which they define.

IV. THE DIVERGENCE FOR THE FAINT STARS

The large difference between the Harvard and Mount Wilson results for the faint stars (see Table III above, or, better, Table IX, *Mt. Wilson Contr.*, No. 97) is to be ascribed to two circumstances: (*a*) the neglect of the distance-correction in reducing the MW plates which enter into the Harvard discussion; (*b*) the application of a correction to the majority of these plates to allow for the order of exposure.

a) Effect of Distance Correction

When the Mount Wilson plates were reduced by Miss Leavitt the values of the distance-correction were not yet available. In

consequence, an attempt was made to avoid the difficulty by including only stars which were within a limited distance of the center of the plate (L, 140). Subsequently some of the plates were reduced by Miss Leavitt, using values of the correction which in the meantime had been determined at Mount Wilson, but without finding any important modification of the results (*loc. cit.*).

The amount of the correction is, however, not negligible. The matter is most easily illustrated in connection with Group 18, as in other particulars these plates were similarly treated in both the Harvard and Mount Wilson reductions. The values for this group given by Miss Leavitt on p. 189 are referred to the zero point of the provisional magnitudes on p. 141 (L, 180); but it is convenient to reduce them to the zero-point defined by the Mount Wilson magnitudes between 10 and 15. When this has been done the difference between Group 18 and the corresponding portion of the Mount Wilson scale will be of the form

$$MW-(18)_H=MW-(18)+D.C.$$

in which $(18)_H$ represents Miss Leavitt's values for Group 18 corrected for zero-point, while (18) is what would have been obtained had the distance-correction been applied. The subtraction of the term D.C. will therefore leave MW−(18), which represents the difference between MW and the Harvard reduction after revision for distance-error.

The zero-point correction is found to be 0.39 mag. and its application to the magnitudes in L, 189, gives the quantities in the second column of Table XI. Values in parentheses are relatively uncertain. The deviations from MW are in the third column; it is seen at a glance that these are very similar to the distance-corrections in the following column, which have been derived from the Mount Wilson reduction tables. The differences between MW and $(18)_H$ corrected for D.C. (in the last column) have a mean value of −0.04 magnitude; but, it is found from Table XII, *Mt. Wilson Contr.*, No. 97, that the weighted systematic deviation of the Mount Wilson reduction of Group 18 (Plates 196, 200, 204) from the final Mount Wilson scale is −0.03 magnitude. The neglected distance-correction, therefore, completely accounts for the difference between

the Harvard and Mount Wilson reductions. Similar differences also affect the plates comprising Groups 19 and 20, but it does not seem necessary to make a detailed comparison. The effect upon

TABLE XI

EFFECT OF DISTANCE CORRECTION

Star	Gr. 18+0.39	$MW-(18)_H$	D.C.	MW−(18)
31	16.35	+6	+7	−1
15s	16.59	−2	+2	−4
32	16.89	−13	0	−13
16s	17.06	−20	+2	−22
33	17.26	−20	−3	−17
17s	17.39	−20	−25	+5
34	17.43	−19	−12	−7
35	17.64	−1	−21	+20
36	17.86	−8	−23	+15
37	18.21	−20	−17	−3
18s	(18.21)	(−27)	−30	(+3)
38	18.44	−24	−26	+2
19s	18.62	−46	−46	0
39	18.67	−9	−28	+19
20s	(19.06)	(−46)	−11	(−35)
21s	18.98	−32	−34	+2
22s	18.93	−18	−15	−3
23s	(19.32)	(−62)	−45	(−17)
24s	(19.25)	(−37)	−25	(−12)
40	19.09	−22	−12	−10
25s	19.34	−50	−16	−34
41	19.32	−30	−12	−18
Means		−19	−15	−4

the scale begins to be appreciable at about the sixteenth magnitude, and gradually increases to a maximum of about 0.4 magnitude for the faintest and least favorably situated stars.

b) Correction for Order of Exposure

From the data used by Miss Leavitt it appears that when several exposures are made upon the same plate, the first and last being short and equal, the images of the first exposure are generally brighter than those of the last. The mean difference she finds to be about a quarter of a magnitude.

The question thus raised is of great importance for the reduction of plates involving successive exposures, for it has a direct and important effect upon the scale. A symmetrical arrangement of

the exposures, or, failing this, a correction depending upon the order of exposure, immediately suggests itself as a means of eliminating the disturbance.

This, however, assumes that the phenomenon develops proportionally to the time, which in the absence of more definite information is a perfectly natural supposition. Miss Leavitt has followed this procedure, and accordingly plates MW 196, 200, 204 (Group 18), for which the arrangement of exposures was symmetrical, received no correction. The plates of Groups 19 and 20 (MW 230, 232, 235, and MW 310, 312, 313), however, were not symmetrical in their arrangement, and have therefore been corrected on the basis of systematic differences derived from the short preliminary and supplementary exposures impressed on each plate.

The values of the measured differences between first and last exposures and the fractional part applied to the images of each of the principal exposures are shown on pp. 122 and 124 of her memoir. As the order of the principal exposures on all of the plates in question was the same, namely, full aperture followed by reduced aperture, the correction always modifies the results in the same direction; the difference in the images with full and reduced aperture is diminished, so that in calculating the scale the faint magnitudes become relatively fainter than they otherwise would have been.

No extensive calculation is required to show how the correction has modified the results from Groups 19 and 20, for a direct indication is afforded by the details in Miss Leavitt's discussion. It was found that the short supplementary exposure on one of the plates (MW 310, H 285) in Group 20 could not be measured, and in consequence no correction was applied (L, 182). The other plates (MW 312, 313; H 286, 287), however, were corrected as usual. Turning now to p. 184 of Miss Leavitt's memoir, it is seen at once that for the faint stars there is a large difference between the mean result for the last two plates and that for the first. For convenience, the deviations of the two scales thus defined from the mean for Group 20 are given in the third and fourth columns of Table XII. The difference between these two series of residuals, which appears in the fifth column, expresses very approximately the effect of the correction.

That the systematic difference thus exhibited is not inherent in the plates themselves appears at once upon referring to Table XII of *Mt. Wilson Contr.*, No. 97, which gives for the Mount Wilson

TABLE XII

CORRECTION FOR ORDER OF EXPOSURE

Star	MW Mag.	Pl. 310−Gr. 20	½(312+313)−Gr. 20	Pl. 310−½(312+313)	Gr. 18−Gr. 20	Gr. 18−Pl. 310	Gr. 18−½(312+313)
31......	16.41	−20	+ 8	−28	−37	−17	−45
15s......	16.57	−21	+ 6	−27	−29	− 8	−35
32......	16.76	−12	+ 5	−17	−33	−21	−38
16s......	16.86	− 6	+ 2	− 8	−23	−17	−25
33......	17.06	−12	+ 4	−16	−19	− 7	−23
17s......	17.19	−20	+13	−33	−22	− 2	−35
34......	17.24	−28	+14	−42	−36	− 8	−50
35......	17.63	−16	+ 9	−25	−39	−23	−48
36......	17.78	−26	+12	−38	−43	−17	−55
37......	18.01	−20	+10	−30	−23	− 3	−33
18s......	17.94	−20	+14	−34	(−10)	(+10)	(−24)
38......	18.20	−50	+25	−75	−49	+ 1	−74
19s......	18.16	−33	+ 8	−41	−57	−24	−65
39......	18.58	−50	+33	−83	−62	−12	−95
20s......	18.60	−40	+27	−67	(−28)	(+12)	(−55)
21s......	18.66	−38	+19	−57	−67	−29	−86
22s......	18.75	−11	+ 8	−19	−87	−76	−95
24s......	18.88	+ 7	− 7	+14	(−36)	(−43)	(−29)
25s......	18.84	−38	+38	−76	−50	−12	−88

reduction (in which none of the plates was corrected) the deviations of individual plates from the mean Mount Wilson scale. For the region covered by the table above we find as a weighted mean:

$$\text{MW reduction: Pl. } 310-\tfrac{1}{2}\text{ (Pl. } 312+313)=+0.02 \text{ mag.}$$

When similarly treated, therefore, the three plates give closely accordant results. As a matter of fact, the effect of the correction is about a tenth of a magnitude greater than the differences shown by the fifth column of Table XII, since the mean distance of the stars from the center of the plate is about 5 mm greater for MW 310 than for 312 and 313; and since none of the Harvard measures were corrected for distance, the magnitudes for MW 310 are relatively too faint by approximately the amount mentioned.

It is of interest further to compare Miss Leavitt's results for MW 310 with those for Group 18. This is at once accomplished by combining the third and sixth columns of Table XII, the results for

the latter column having been derived from L, 189. The differences in the seventh column reveal a relatively small systematic effect. In other words, the uncorrected plate MW 310, which presumably is affected by the phenomenon under discussion since all of the other Mount Wilson plates of unsymmetrical arrangement seem to be so affected, gives results agreeing closely with those for Group 18 from which the systematic effect has supposedly been eliminated by a symmetrical arrangement of the exposures. On the other hand, plates MW 312 and 313, which have received correction, differ widely in their results from those of Group 18 (last column Table XII).

This immediately raises a suspicion as to the validity of the assumption upon which the above-described procedure was based. The matter can be tested further by means of the Mount Wilson results for individual plates, part of which appear in Table XII of *Mt. Wilson Contr.*, No. 97. Of the plates there listed Nos. 230, 232, 235, 310, 312, 313, and 314, on the basis of the procedure followed by Miss Leavitt, would require correction for the order of exposure. None of the others, and none of the fifteen plates of shorter exposure which were also used in deriving the MW scale, would need such correction, either because of a symmetrical arrangement of the data or because of the minuteness of the differences between the first and last exposures (in the case of Nos. 769 and 808). A comparison of the scales from the two groups of plates should therefore show at once the result of having neglected the correction in the case of the questionable series. Moreover, the difference in the two scales should be a reliable indication of the effect produced, for the amount of the material is such as to reduce to a minimum the errors arising from other sources.

From Table XII, *Mt. Wilson Contr.*, No. 97, we find the weighted deviations of the mean scale for Plates 230, 232, etc., from the final Mount Wilson scale to be as shown in Table XIII. Since the weight of the results from the plates of symmetrical arrangement is considerably in excess of that from Nos. 230, 232, etc., the scale difference for the two groups is somewhat less than twice the systematic difference in Table XIII; but the result thus attained is within the limit of error which may arise from other sources, and we are led

again to the conclusion expressed above, namely, that the assumption upon which the whole procedure is based is unjustifiable.

TABLE XIII

MW—(PL. 230, 232, ETC.)

MW Mag.	Syst. Dev.	Wt.	MW Mag.	Syst. Dev.	Wt.
11.1........	−0.005	30	16.1........	−0.018	154
12.2........	+ .053	39	16.6........	− .029	186
13.2........	+ .031	52	17.1........	− .005	159
13.7........	+ .015	68	17.6........	+ .016	233
14.1........	− .028	61	18.1........	+ .016	241
14.6........	− .001	134	18.6........	+ .040	358
15.1........	− .024	174	19.1........	+0.056	77
15.6........	−0.019	127			

Moreover, we may say with some definiteness that the questionable group of plates requires no appreciable correction whatever in order to arrive at a correct scale (excluding unsuspected disturbances which may arise from other sources). Both our own measures and those by Miss Leavitt and Miss Leland show that in the main there is no sensible difference between the first and last exposures upon plates MW 196, 200, and 204. Plates 769 and 808 which were used only for the Mount Wilson scale give a similar result. The mean scale from these five plates should therefore be free from any influence due to the phenomenon under discussion; but this scale we find from Table XII, *Mt. Wilson Contr.*, No. 97, to be practically identical with that derived without correction from the plates which do show differences between initial and final exposures. Apparently, therefore, the principal exposures on these plates, which are the ones used in deriving the scale, contain little or nothing of the systematic difference that is revealed when the short preliminary and final exposures are compared.

The phenomenon is obviously photographic in origin, and Mees has suggested,[1] as the most probable explanation, that it is due to a depression of the sensitiveness of the plate through the absorption of water-vapor upon exposure to the air. No data are available as to the time required for a condition of equilibrium to be established, but it is perhaps not inconceivable that the major part of the change

[1] *Astrophysical Journal*, **40**, 236, 1914.

should occur within a comparatively short interval after the beginning of the exposure. It might thus easily happen that only the first of a series of equal exposures would be markedly different from the others; and if the exposures were long the difference might be more or less completely obscured.

The question will be discussed further at another time; but one point requires mention before the subject is dismissed, and that is the apparent susceptibility of the measures used in this part of the discussion to systematic error. This is well illustrated by the results for different observers and different scales given in L, 122, and again, by the comparison of these results with our own for the same plates as measured by Miss High and Miss Richmond.

A striking result is the fact that measures made here agree in fixing a much smaller value for the mean difference between the initial and final exposures than that found by Miss Leavitt; but most extraordinary is the circumstance that we find the variation of the difference with size of image to be entirely different in character from that illustrated in Miss Leavitt's memoir (Plate 3, Fig. 1). For the Mount Wilson plates she finds in general that the difference is a maximum for the faintest images, while for these we usually find no difference at all.

Had we corrected the Mount Wilson plates, using our own measures, the scale for the six or seven plates thus modified would have been deflected by rather less than half the amount entering into the Harvard results. The matter would be disconcerting were it not pretty clear that it should not enter into the discussion at all, and were it not further clear that, when similarly treated, Miss Leavitt's measures of the more essential portion of the data and our own give practically identical results for the scale.

The preceding discussion does not extend the comparison with the Harvard results below the nineteenth magnitude; it is unnecessary, however, to carry the matter further. The Harvard magnitudes of the faintest stars depend upon an extrapolation of the scale for the region just considered, and it is evident that the divergence which has entered in the manner indicated must produce even greater differences for still fainter objects. These are quite sufficient to account for the difference in the limiting magni-

tudes reached in the two investigations—21 for Harvard and 20 for Mount Wilson.

SUMMARY

1. The Mount Wilson color-indices of the Polar Sequence stars afford the possibility of investigating the influence of color upon the results obtained with the various instruments used in deriving the Harvard photographic scale. It is found that corrections of a few hundredths of a magnitude (4th col., Table III) are required to reduce this scale (that of *H.C.*, No. 170+0.08 mag.) to a homogeneous color-system (HH, Table III).

2. The comparison of the Harvard Homogeneous Scale (HH) with MW reveals a color-equation with a coefficient of 0.06. Including the scale-divergence for the bright stars, we have to the sixteenth magnitude, approximately

$$\mathrm{MW}-\mathrm{HH}=+0.061\ (\mathrm{HH}-6.0)+0.06\ \mathrm{C}$$

in which C is the color-index, and in which, further, the first term is constant and equal to +0.24 magnitude from the tenth magnitude onward. The allowance for color has therefore appreciably diminished the divergence between the sixth and the tenth magnitudes given by the earlier comparisons. For the faint stars there remains a large difference between the two scales, which at the twentieth magnitude (MW) amounts to about one magnitude.

3. The acceptance of the HH magnitudes between 10 and 15 as an accurately established scale makes it possible to re-reduce much of the Harvard data for the bright stars by methods which are free from various uncertainties that necessarily affect the original reduction. It is thus found that the HH magnitudes apparently require small corrections which improve the agreement with MW and extend the region of parallelism for the two scales upward from the tenth to the eighth magnitude (6th and 9th cols., Table X). The remaining discrepancy of a quarter of a magnitude is thus thrown upon the stars which are brighter than the eighth magnitude.

4. The divergence for the faint stars is due to the neglect of the distance-correction in reducing the Mount Wilson plates used for the Harvard scale, and to the application of a correction intended

to remove the influence of the systematic difference sometimes affecting initial and final exposures on photometric plates. With proper allowance for these differences in procedure, the Harvard scale becomes parallel with that of Mount Wilson in the region of the faint stars.

5. The photographic phenomenon which shows itself as a systematic difference between first and last exposures seems for the most part to occur within a short interval after the beginning of the first exposure, for the principal exposures, on the Mount Wilson plates at least, require no correction in order that they may give results accordant with those from plates which do not show the effect at all.

MOUNT WILSON SOLAR OBSERVATORY
February 9, 1915

MISCELLANEOUS NOTES ON VARIABLE STARS[1]

By HARLOW SHAPLEY

The following collection of notes and data on subjects related almost exclusively to variable stars represents the results of short or partial investigations that have been undertaken during the last two years. A part of the observational work involved was done at the Princeton University Observatory, but the reduction and discussion of the observations and the statistical and other computations were made at Mount Wilson. The incomplete series of observations on eclipsing variables are published now because there will be no immediate opportunity to continue the work.

The contents of the several notes are as follows:

1. Comparison of the observed solar darkening with the assumed stellar darkening—an inquiry into the applicability of the formula $J=J_0\ (1-x+x \cos \gamma)$ to the representation of the distribution of light over the disk of the sun. The investigation is based on published and unpublished observations from the Astrophysical Observatory of the Smithsonian Institution.

2. Observations of a minimum of the F-type eclipsing star U Pegasi and the derivation of new light-elements.

3. Photometric measures of *o* Persei for the purpose of testing its light-variability.

4. Photometric observations of R Canis Majoris.

5. The visual range of AE Cygni, with a provisional solution for uniform orbital elements.

6. The relation between spectrum and length of period for close double stars, including eclipsing variables and spectroscopic binaries of the non-Cepheid type.

7. On the number of naked-eye variable stars of different classes.

The manuscript data concerning recent measures of the distribution of brightness over the disk of the sun were kindly furnished

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 99.

by Mr. Abbot. The sixth and seventh notes were prepared and written by Mrs. Shapley.

I. ON DARKENING TOWARD THE LIMB OF THE SUN

In the study of the light-curves and orbital elements of eclipsing binaries the nature of the distribution of brightness over the apparent stellar surfaces has been found to be of paramount importance. The observed darkening toward the limb of the sun was taken as a basis for the assumption as to the general character of stellar darkening. Since the adoption of an empirical cosine formula to represent the diminution of light from center to limb, the actual existence of darkening for stars of various spectral types has been definitely demonstrated;[1] but the suitability of this particular formula remains undetermined, and the dependence of the darkening coefficient on wave-length and spectral type is as yet unknown.

In the case of the sun the applicability of the adopted formula, and also the dependence of the coefficient on the mean wave-length of the light used in obtaining the light-curve, can be directly studied with the aid of the bolometric observations of the Astrophysical Observatory of the Smithsonian Institution. For stars of other spectral types, however, we can now only speculate as to how the law of darkening may differ and what the degree of darkening may be for the light of different wave-lengths.[2] There is perhaps an indication that it is greater for the bluer stars, and a refined study of appropriate light-curves will doubtless contribute something to the solution of the problem.

A comparison of the bolometric results with the formula has not been made heretofore, and in the present note a brief discussion will be given of the diagrams, Figs. 1 and 2, in which such a comparison is presented.

Letting J_0 be the value of the apparent surface brightness J at the center of the disk, the assumed relation between J and the

[1] *Contributions from the Princeton University Observatory*, No. 3, pp. 106–110, 1915.

[2] Relative to the uncertainty in orbital computations arising from the present indefinite state of the darkening-at-the-limb problem see the above-cited work, chap. iii, and *Astrophysical Journal*, **40**, 219, 1914.

apparent distance from the center is $J/J_0 = 1 - x + x \cos \gamma$, where the darkening coefficient is x and γ is the angle between the line

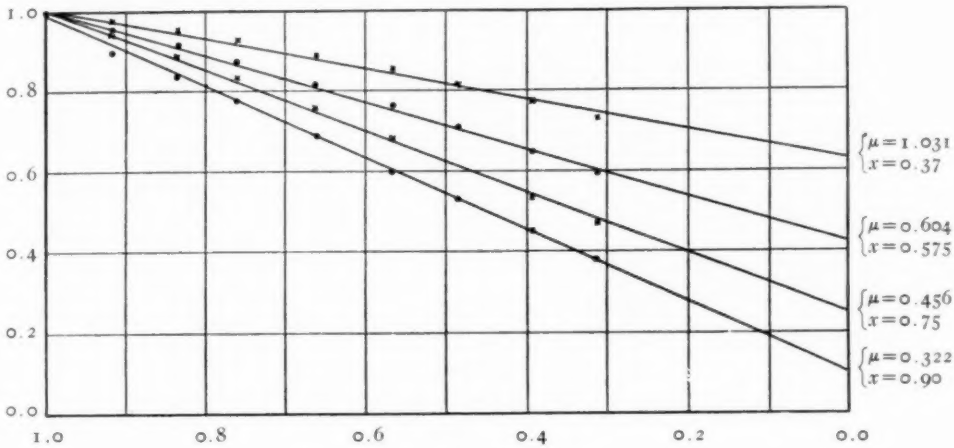

FIG. 1.—Solar darkening in 1907
Abscissae: cos γ; Ordinates: J/J_0

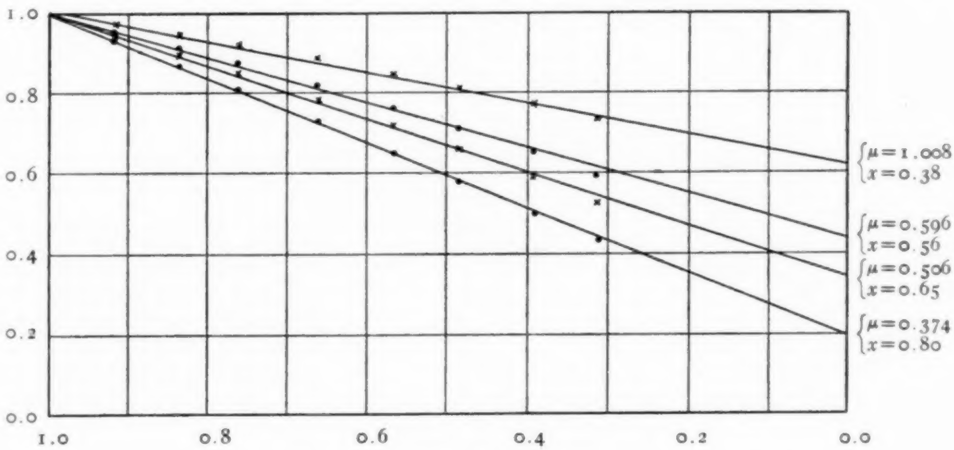

FIG. 2.—Solar darkening in 1913
Abscissae: cos γ; Ordinates: J/J_0

of sight and the surface normal. The means of the bolometric measures made at Washington in the year 1907 (sun-spot maximum)

are given in *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **3**, 157, 1913, and an accompanying diagram represents sin γ plotted against J/J_0 for several different wave-lengths. When we plot the measures against cos γ, as I have done for four representative wave-lengths in Fig. 1, the observed points should lie on a straight line if the foregoing formula is sufficient. The bolometric work at Mount Wilson during September, October, and November 1913 (sun-spot minimum) is represented for four wave-lengths in Fig. 2. In both diagrams the mean wave-length of the part of the spectrum used is written opposite the straight line that most nearly satisfies the observations, and the slope of the line, which is the darkening coefficient x, is also given. $1-x$ is the brightness at the limb in terms of the central brightness. The data for $\mu=0.322$ are from photographic results by Schwarzschild and Villiger.[1]

Three results of importance to this inquiry can be deduced from the diagrams:

1. The observed points, though lying on smooth curves, do not define straight lines for any of the wave-lengths of either series, and the deviations are evidently larger than the uncertainties of the observations. Therefore the formula adopted for the stellar investigation does not completely represent the observed solar darkening. The representation is quite satisfactory, however, since the error is generally less than 1 per cent and scarcely exceeds 3 per cent for any wave-length or for any distance from the center up to 95 per cent of the radius (cos $\gamma=0.31$). Beyond this point there is an indication that the brightness may fall off rapidly; but, to quote Abbot's *The Sun*, p. 107, footnote, "There is a tendency of all the data to show a less rapid fall of brightness from 95 to 97 per cent out on the radius than would be expected. This may be due to error." The effect on the light-curve of a variable star of the error in the assumed formula would be entirely negligible (even if we had an exact value of the coefficient) over at least 95 per cent of the curve at minimum. We conclude, therefore, that the adopted cosine law of darkening, though manifestly inexact, is a very good approximation and sufficiently accurate for the

[1] *Astrophysical Journal*, **23**, 284, 1906.

stellar work of the present time. Knowledge of the amount of darkening for a given wave-length and for different spectral types is a more important and immediate need.

2. For the solar spectral type, at least, the value of the darkening coefficient is conspicuously greater for photographic light-curves than for visual—a well-known phenomenon. The average value of x is approximately 0.75 for the mean effective photographic light, and for the mean visual part of the spectrum, about 0.6. This of course does not necessarily mean that the coefficient would be greater for stars of bluer spectral types.

3. For a given wave-length the darkening coefficient decreases with the number of sun-spots, this change being more pronounced for the shorter wave-lengths. Or stated in another way,[1] "there was distinctly less contrast of brightness between the edge and center of the sun's disk in 1913 than in 1907," and the shorter wave-lengths change in contrast more than the longer. This can be shown easily by plotting μ against x for the two series. There are also changes in contrast accompanying the short-period irregular variations of the sun. Apparently, then, our problem may involve a variable darkening coefficient.

2. NEW LIGHT-ELEMENTS FOR U PEGASI

On the basis of the following series of observations made with the polarizing photometer during a single principal minimum of U Pegasi it is possible to make a small but definite correction to the period. The star B.D.$+14°5077$ (7.9), which is about three magnitudes brighter than the variable, was used for the comparisons.

The light-elements of U Pegasi generally adopted at present are those deduced by Roberts[2] from Wendell's observations at the Harvard Observatory:[3]

$$\text{Prin. Min.} = \text{J.D. } 2415021.2367 + 0^{\text{d}}.374784 \text{ E}$$

Apparently a small error exists in the yearly *Vierteljahrsschrift* catalogues since 1912 in giving the light-elements and ephemeris

[1] *Report of the Secretary of the Smithsonian Institution for the Year Ending June 30, 1914*, p. 94.

[2] *Monthly Notices*, **66**, 135, 1906.

[3] *Harvard Annals*, **69**, 53, 1909.

for this star.[1] The initial epoch there reads J.D. 2415021.2469; hence all times are predicted 15 minutes too late, which is a noticeable amount in this case of a nine-hour period.

TABLE I

PRIMARY ECLIPSE OF U PEGASI, OCTOBER 31, 1913

Gr. Helioc. Mean Time	Phase	Magnitude Difference	O−C
			mag.
$14^h\ 3^m$	$-0^h 13^m$	3.37	−0.04
14 13	−0 3	3.46	+ .01
14 25	+0 9	3.39	− .04
14 35	+0 19	3.35	− .01
14 46	+0 30	3.31	+ .06
14 59	+0 43	3.17	+ .03
15 19	+1 3	3.00	− .02
15 29	+1 13	2.98	.00
15 39	+1 23	2.87	− .07
15 49	+1 33	2.91	.00
16 1	+1 45	2.87	.00
16 12	+1 56	2.90	+0.04

The corrected heliocentric time of principal eclipse on October 31, 1913, predicted by means of Roberts' elements, is 13^h48^m, G.M.T. The observed time was 14^h16^m, G.M.T., with an uncertainty not exceeding two or three minutes. The difference of 27 minutes represents the accumulation of the error in the period during sixteen years. The accuracy of the elements determined by Roberts is sufficient to leave no doubt as to the number of intervening periods. There were 13,478 revolutions between the date of my observations and the published initial epoch, and about two thousand more between that arbitrary epoch and the middle of the interval over which Wendell's observations of the principal minimum extend. The correction to the period derived from these data is +0.10 seconds, so that the revised value is $0^d.3747852$, with a probable uncertainty of two or three units in the last place. The new heliocentric epoch of primary minimum is

$$\text{J.D. } 2420072.5754 = 1913,\ \text{October } 31,\ 13^h48^m.6,\ \text{G.M.T.}$$

The phases given in the table above were computed by means of the new elements. The residuals in the last column show the

[1] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **47**, 301, 1902.

agreement of the present short series of measures with the theoretical darkened curve based on Wendell's observations.[1] The representation is quite satisfactory, both as to depth and as to duration of the eclipse, and suggests that no marked change has occurred in the nature of the eclipse during the last fifteen thousand revolutions of the components. The data here available are obviously insufficient, however, to give this last conclusion much weight; but it may be pointed out that such extremely short-period systems as U Pegasi and W Ursae Majoris are deserving of careful and continuous attention from observers, with the object of establishing the presence or absence of sensible perturbations in the length of the period, the range of variation, or the general nature of the light-curve. Both systems are very close, their orbits are apparently circular, their spectral types are F and G, respectively, their components are the densest stellar bodies known, and both possess a high degree of ellipticity. In fact, one-third of the range of light-variation of U Pegasi at principal minimum is attributable to the rotation of the elongated components.

3. PHOTOMETRIC OBSERVATIONS OF *o* PERSEI

The possibility that the spectroscopic binary *o* Persei is also an eclipsing variable has been recognized by several observers, and the constancy of its light has been put to test by Lau,[2] Guthnick,[3] and Hertzsprung.[4] Both Lau and Guthnick announce a variability of the light through a range of 0.35 mag., and the latter finds the time of minimum in agreement with the expectation of eclipse. Hertzsprung, on the other hand, finds no definite evidence of variability, but he points out that none of his observations were made within five hours of the predicted time of principal minimum. Before

[1] *Contributions from the Princeton University Observatory*, No. 3, p. 176, *et passim*, 1915.

[2] *Astronomische Nachrichten*, **196**, 427, 1914.

[3] *Ibid.*, **197**, 303, 1914. Since writing the above, Guthnick's discussion of his preliminary results for this star with the photo-electric photometer have been received (*Veröffentlichungen der Königlichen Sternwarte zu Berlin-Babelsberg*, **1**, No. 1, p. 57, 1914). The results, which are not as yet very definite, suggest a variation of a tenth of a magnitude.

[4] *Astronomische Nachrichten*, **199**, 140, 1914.

learning that the star was under investigation elsewhere the writer had commenced a series of observations with the Princeton polarizing photometer. The results of measures on three nights are given in Table II.

TABLE II

OBSERVED MAGNITUDES OF *o* PERSEI

Date	Gr. Geoc. M.T.	Phase from Super. Conj.	Magnitude Difference	Remarks
			mag.	
1913, Nov. 21 ...	13^h31^m	$+19^h27^m$	$a-o=+4.01$	
	13 44	+19 40	$a-o=+3.99$	
	13 55	+19 51	$a-o=+4.03$	
Dec. 12 ...	20 9	− 0 12	$b-o=+4.62$	
	20 24	+ 0 3	$b-o=+4.67$	
	20 40	+ 0 19	$a-o=+4.00$	
	20 55	+ 0 34	$a-o=+4.08$	Measure interrupted
	21 18	+ 0 57	$a-o=+3.99$	Star low
Dec. 15 ...	10 26	+62 5	$a-o=+3.90$	Moon
	10 36	+62 15	$a-o=+3.90$	
	10 50	+62 29	$b-a=+0.65$	

The phases were computed with the aid of the spectroscopic elements by Jordan.[1] The comparison stars are a=B.D.+31°644 (8.3), and b=B.D.+31°643 (8.2). Each observation is the mean of 16 comparisons and has a probable error of approximately ±0.03 mag. The fourth observation on December 12 should perhaps be given low weight.

The magnitudes in the table, so far as they go, give no indication of variability, although they were made apparently at the most favorable times. If, however, the orbit which was found by Jordan to be circular should be slightly eccentric, the principal minimum might be displaced sufficiently from the predicted time to render inconclusive the foregoing evidence against variability.

4. PHOTOMETRIC OBSERVATIONS OF R CANIS MAJORIS

The remarkable hump on the light-curve of R Canis Majoris at the end of the principal minimum was measured independently by Professors E. C. Pickering and O. C. Wendell at Harvard more than twenty years ago. The existence of the phenomenon

[1] *Publications of the Allegheny Observatory*, **2**, 63, 1911. The period is 4.41916 days.

at that time seems to be entirely beyond question; but so far as I know there has been no recent observation of the star, and no further investigation of the anomalous feature. With the hope of throwing some light on this peculiarity, a series of observations was undertaken with the Princeton polarizing photometer, but because of the southern declination of the star and the continued unfavorable weather during the short observing season, the work was given up before satisfactory results could be obtained. The observations given below may be of value to other observers, however, not only in showing that if the hump now exists it has perhaps shifted along the curve away from minimum, but also in showing that the light-elements by Chandler do not at present adequately represent the observed times of minima.

The predicted time of eclipse on March 22, 1913, was $15^h 32^m$, Gr. Hel. M.T. The observed time of mid-eclipse was about half an hour earlier. This difference may not indicate that a correction to the mean period is necessary. The orbit of R Canis Majoris

TABLE III

OBSERVED MAGNITUDES OF R CANIS MAJORIS

Date	Gr. Geoc. M.T.	$a-v$	Date	Gr. Geoc. M.T.	$a-v$
1913		mag.	1913		mag.
Mar. 12	$11^h 48^m$	2.11	Mar. 22	$12^h 59^m$	2.02
	11 57	2.06		13 8	2.04
	12 14	2.06		13 26	1.90
	12 24	2.14		13 36	1.88
	12 40	2.10		13 47	1.88
	12 49	2.10		14 3	1.77
	13 2	2.14		14 13	1.66
	13 11	2.12		14 30	1.52
	13 23	2.09		14 44	1.49
	13 33	2.10		14 54	1.42
	13 43	2.19		15 4	1.37
	13 57	2.18		15 16	1.43
Mar. 22	12 37	2.12		15 30	1.43
	12 50	2.02		15 44	1.56

has an exceptionally high eccentricity for an eclipsing binary;[1] furthermore, the line of apsides is evidently in fairly rapid motion, but with an unknown period.[2] The resulting oscillation in the

[1] *Publications of the Allegheny Observatory*, **3**, 52, 1913.

[2] *Contributions from the Princeton University Observatory*, No. 3, p. 95, 1915.

time of the principal minimum may then easily be of the order of the discrepancy observed. Further investigation of the light-curve is much to be desired, not only to contribute to our knowledge of orbital changes in close binary systems of pronounced eccentricity, but also to elucidate the meaning and the present nature of the hump in the light-curve, which stands out as the most conspicuous unexplained irregularity known in the light-curve of any eclipsing binary.

The comparison star used in the measures above, a = B.D. $-16°1886$ (8.0), is reddish. The last two observations of each night should receive low weight because of the low altitude and trouble with smoke and haze.

The observations on March 22, 1913, indicate that the range is at least a tenth of a magnitude greater than observed by Wendell fourteen years earlier. The results of computations on the photometric orbit and a discussion of them are given in *Contributions from the Princeton University Observatory*, No. 3.

5. NOTE ON THE PROVISIONAL ORBIT OF AE CYGNI

The following series of observations of the faint eclipsing variable AE Cygni are of value chiefly in showing the visual range of variation and in giving one epoch of minimum. They are not conclusive as to the existence of a secondary minimum nor of ellipticity. The comparisons were made with the star a = B.D. $+29°4347$ (8.2). Each observation in Table IV contains 16 settings made with the polarizing photometer.

TABLE IV

OBSERVATIONS OF AE CYGNI

Date	Gr. Geoc. M.T.	$v-a$	Date	Gr. Geoc. M.T.	$v-a$
1912		mag.	1912		mag.
June 10	$16^h 53^m$	+3.04	Nov. 3	$15^h 15^m$	+2.40
	17 6	3.17		15 29	2.42
	17 23	3.16		15 48	2.40
	17 38	3.21		16 1	2.44
			1913		
Nov. 3	14 16	2.54	Jan. 1	11 11	2.39
	14 31	2.42		11 25	2.35
	14 48	2.46		11 47	2.46
	15 1	2.42		12 2	2.40

The light-elements given by Williams[1] represent the measures printed in Table IV with sufficient accuracy. His light-curve, however, which is based on 83 visual observations, presents unusual difficulties in the derivation of even approximate orbital elements. In order to investigate the system further my observations were undertaken, but the faintness of the star and the inconvenient length of the period ($23^h 15^m.6$) have prevented decisive results from being obtained. The range of variation according to my measures is 0.77 mag. With this value the scale-readings given by Williams were converted into magnitudes and the resulting light-curve investigated. If the period is as given above, the provisional orbital elements are:

Ratio of radii	=0.75
Inclination of orbit	=85°
Radius of orbit	=1.00
Radius of larger star	=0.45
Light of larger star	=0.50

But in this case a secondary minimum of three-tenths of a magnitude would be required, the existence of which is not verified by my observations; moreover, the components would be so close together that considerable elongation would be expected and the range as given above would then be illusory for orbital computations.

It seems more probable that the period is double the value given by Williams, that the stars are not markedly ellipsoidal, and that the alternate minima are probably of unequal depth. As a consequence, the components would be found to be of nearly equal dimensions, much smaller relative to the size of the orbit than given above, and the mean density of the system would be of the order of two-tenths that of the sun. If the shorter value of the period is correct, the mean density is one-half as large. The spectral type is unknown.

6. ON THE PERIODS AND SPECTRA OF CLOSE BINARY STARS

The following study has been made to supplement the recent work of Wicksell on the frequency of the periods of spectroscopic

[1] *Astronomische Nachrichten*, **184**, 97, 1910.

binaries,[1] and to determine whether the conclusions reached in his paper can be considered more than an apparent result depending on selection. He finds two distinct maxima in the frequency-curves for all spectral types and attempts to explain this on the assumption that in spectroscopic double stars the groups of long and short periods depend on two different principles or conditions of origin. He suggests, however, the possible insufficiency of his data.

Since there is essentially no difference between spectroscopic and eclipsing binaries, except that the latter are limited in the possible values of their orbital inclination, there is no reason why the data for the two classes of stars should not be combined in a study of this kind. Accordingly a card catalogue of eclipsing variables was made, which contains over 200 entries, and the 121 stars for which both period and spectral type are known form, together with Wicksell's data, the basis of this discussion. From Wicksell's list all those spectroscopic binaries which are known also to be eclipsing variables were removed; 14 spectroscopic binaries not in his list, but whose periods and spectral types are available, were added; and for one star, π_5 Orionis, a later value of the period was substituted. All Cepheid variables were excluded, since it is not certain that they are binary systems; hence four stars not called Cepheids by Wicksell were dropped from his list—β Cephei and α Ursae Minoris, two known Cepheids, and β Canis Majoris and λ Andromedae, whose orbits suggest that they also may belong to this class. A separate study was then made (1) of the eclipsing variables, (2) of the spectroscopic binaries not also known to be eclipsing binaries, and (3) of both the first and second groups combined.

Table V shows the total number of eclipsing and spectroscopic binaries of each spectral type in equal intervals of the logarithm of the period. Two Ma stars and one of type Oe5 were not considered.

Frequency-curves, similar to Wicksell's, were drawn for each spectral type in each of the three above-named groups, i.e., eclipsing binaries, spectroscopic binaries, and the total of both. Curves were

[1] *Arkiv för Matematik, Astronomi och Fysik,* **10,** No. 6, 1914.

also drawn for each group, using all the stars regardless of spectral type, and these are reproduced in the accompanying diagrams.

The supposed secondary maximum is found to occur definitely only in the case of the B-type stars in the spectroscopic group; and in that group it is due solely to six long-period stars. If, as shown by the dotted lines in the diagrams, all B-type stars are omitted, the phenomenon of the two maxima completely disappears. The same result is obtained by omitting only the six B stars of long period. These anomalous systems are: κ Velorum, $116\overset{d}{.}65$; ϕ Persei, $126\overset{d}{.}6$; ν Orionis, $131\overset{d}{.}3$; ζ Tauri, 138^d; π Andromedae, $143\overset{d}{.}67$; and μ Sagittarii, $180\overset{d}{.}2$. ϕ Persei and μ Sagittarii are classified as having peculiar spectra. Evidently then the real problem lies in explaining the spectra or periods of these six stars. The next longest period among the B's is 31 days.

TABLE V

LOGARITHM OF PERIOD AND SPECTRAL TYPE

Spectrum	−0.8 to −0.4	−0.4 to 0.0	0.0 to 0.4	0.4 to 0.8	0.8 to 1.2	1.2 to 1.6	1.6 to 2.0	2.0 to 2.4	2.4 to 2.8	2.8 to 3.2	3.2 to 3.6	3.6 to 4.0	Total No. of Binaries
B	....	....	9	16	10	6	....	6	....	....	....	....	47
A	....	9	26	40	14	9	3	2	....	1	1	....	105
F	1	3	3	8	3	....	3	1	2	....	1	2	27
G	1	....	....	2	2	2	1	2	2	3	....	....	15
K	....	....	....	....	....	2	....	2	1	....	....	....	5
Total ..	2	12	38	66	29	19	7	13	5	4	2	2	199
F-G-K	2	3	3	10	5	4	4	5	5	3	1	2	

Thus, after more than doubling the data for close binary systems, it appears that the secondary maximum in length of period does not exist, although there are slight indications of it in some of the curves for the separate spectral types. These, however, disappear in the totals except for the six stars mentioned. It is found also that the maximum of all the frequency-curves falls between the values 0.4 and 0.8 for the logarithm of the period, that is, for the periods between approximately 2.5 and 6 days. The B-type stars seem to have a comparatively shorter range in length of period than any of the others (except the K stars, for which the data are too meager to justify any conclusions), namely, 1.45 to

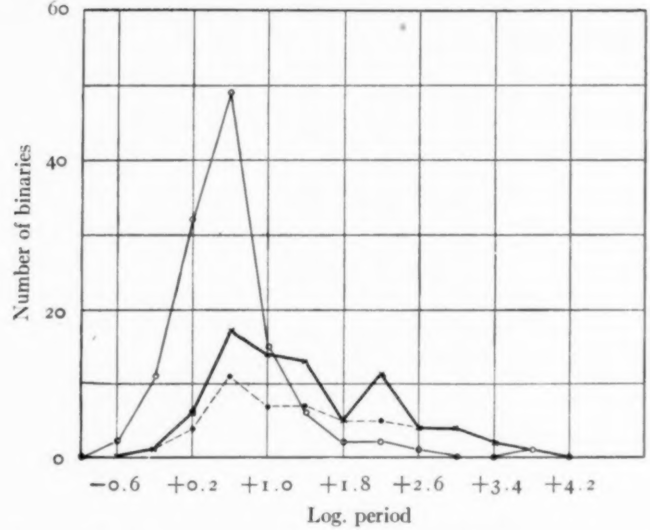

FIG. 3.—Frequency-curves: eclipsing variables, light line; spectroscopic binaries, heavy line; spectroscopic binaries excluding B-type spectra, broken line.

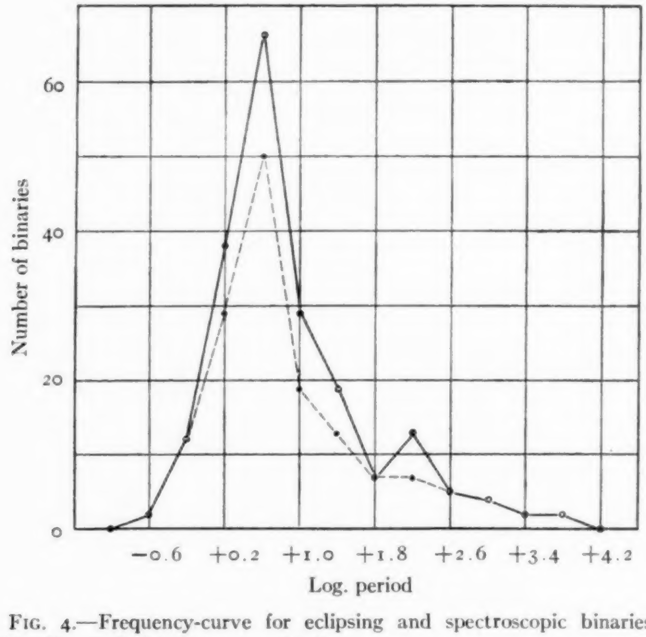

FIG. 4.—Frequency-curve for eclipsing and spectroscopic binaries combined (broken line excludes B-type spectra).

180.2 days; or, without the stars that may be anomolous, 1.45 to 31 days. For the systems with second-type spectra, in which the F's predominate, the range is the greatest, being 0.33 to 9905 days. It is of interest to note that one-third of the second-type systems have periods greater than 180 days and one-seventh shorter than 1.45 days, the outside limits given above for the binaries with B-type spectra.

7. ON THE NUMBER OF NAKED-EYE VARIABLE STARS

As a result of his search for variable stars in globular clusters, Bailey found that about 2.5 per cent of all the stars examined show distinct light-variation; this percentage was compared with the number of variables among the naked-eye stars, which he placed at 1 per cent.[1] Since that study was made the number of known variable stars has considerably increased. In order to get an idea of the percentage of naked-eye stars now known to be variable, and to see how they are distributed in the different variable star classes for each interval of magnitude, the data in Table VI were collected.

TABLE VI

Magnitude	Long Period	Short Period	Eclipsing	Miscellaneous	Total Variables	Total Stars	Percentage Variable
< 1.0 ..	0	0	0	5*	5	12	17 per cent*
1.0 to 1.9 ..	0	0	1	1	2	28	7
2.0 to 2.9 ..	1	1	5	.1	8	105	8
3.0 to 3.9 ..	3	6	4	7	20	275	7
4.0 to 4.9 ..	5	4	4	9	22	750	3
5.0 to 6.0 ..	18	7	4	20	49	2160	2
Total ..	27	18	18	43	106	3330	3

* Three of the five variables of this magnitude are Novae, and, as they are now from the ninth to the twelfth magnitude in brightness, they are not included in the percentage.

The second, third, fourth, and fifth columns give the number of variables belonging to the different classes named, whose brightness at maximum light corresponds to the magnitude tabulated in the first column. In the class "Miscellaneous" are included variables with unknown or irregular periods and the Novae. Column six contains the total number of variables, and seven the total

[1] *Harvard Annals*, **38**, 1, 1902.

number of all stars, while the final column gives the percentage of variables in each magnitude division. It is seen that 3 per cent of all stars visible to the naked eye are known to vary in light.

In collecting this material the many bright stars recently suspected of small variation by Lau[1] and Guthnick[2] have not been considered, although many of them will probably be admitted soon to the lists of known variables. Had they been included the total percentage would be increased from three to more than five.

Mount Wilson Solar Observatory
February 1915

[1] *Astronomische Nachrichten*, **196**, 427, 1914.

[2] *Veröffentlichungen der Königlichen Sternwarte zu Berlin-Babelsberg*, **1**, No. 1, 1914; *Astronomische Nachrichten*, **191**, 169, 1912.

SOME ORBITAL CHARACTERISTICS OF THE CEPHEID-GEMINID VARIABLES AND A SUGGESTED EXPLANATION OF THE CAUSE OF THEIR LIGHT-VARIATIONS

By C. D. PERRINE

As one result of the investigation of the peculiar distribution of the Cepheid-Geminid variables, and having in mind some of their other peculiarities, I was led to examine the orbits which had been computed. Several seemingly well-marked and suggestive characteristics appeared, in addition to the small values of $a \sin i$ and $\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$ to which Campbell refers.[1] These are: (1) generally large orbital eccentricities; (2) considerable resemblance in ω; (3) considerable uniformity in K. To these may be added: (4) very small, and sensibly common, proper motions; (5) the well-known fact that these stars all belong to classes F and G; and (6) their strong preference for the Milky Way. The data above referred to are brought together in Table I.

There appears to be some relation between the eccentricities and the angles of periastron as shown by Table II.

Why the first four do not agree with the others more closely is not clear. In these orbits (Table II) the larger the eccentricity the more nearly in general the angle ω approaches 90°.

The tendency of the last nine of the stars of Table II to have their angles of periastron near to 90°, or, in other words, the major axes of their orbits near to the line of sight in at least one plane, is undoubted. It may be that this is a preference of the plane of their orbits for the Milky Way. With a better knowledge of the distance and mass of the Milky Way as well as of the stars, investigation of this point might be very fruitful.

It may be noted in passing that in over two-thirds of all of the spectroscopic orbits available to date the value of ω lies between

[1] *Lick Observatory Bulletin*, No. 181, **6**, 51, 1910.

TABLE I

H.R.	Mag. Max.	Star	α 1900	δ 1900	P	Proper Motions* α	Proper Motions* δ	ω	e	K	$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$	$a \sin i$	V
										km		km	km
	7.2	SZ Tauri	$4^h 31^m 4$	$+18° 20'$	$3^d 15$	$+^s.0066$	$-''.012$	$76°.7$	0.24	10.9	0.0004	460,000	−15.2
2332....	5.0	RT Aurigae	6 22.1	+30 34	3.73	− 5	− 23	95.0	.37	18.0	.0018	856,500	+12.2
2650....	3.7	ζ Geminorum	6 58.2	+20 43	10.15	− 3	− 8	333.0	.22	13.2	.0023	1,797,800	− 4.7
6616....	4.0	X Sagittarii	17 41.3	−27 48	7.01	− 4	− 22	93.6	.40	15.2	.0016	1,334,000	− 3.2
6661....	5.8	Y Ophiuchi	17 47.3	− 6 7	17.12	+ 17	− 30	201.7	.16	7.7	.0011	1,999,000	+10.8
6742....	4.8	W Sagittarii	17 58.6	−29 35	7.60	+ 7	− 12	70.0	.32	19.5	.0050	1,930,000	−18.8
6863....	5.8	Y Sagittarii	18 15.5	−18 54	5.77	+ 5	− 11	32.0	.16	19.0	.0040	1,485,000	+16.8
7518....	6.7	SU Cygni	19 40.8	+29 1	3.84	....	....	346	.21±	25 ±	.0058±	1,350,000±	−15.4
7570....	3.5	η Aquilae	19 47.4	+ 0 45	7.18	+ 5	− 9	68.9	.49	20.6	.0043	1,773,000	+ 1.0
7609....	5.6	S Sagittae	19 51.5	+16 22	8.38	+ 2	− 3	70±	.35±	19 ±	.0049±	2,000,000	+ 4.4
7988....	5.5	T Vulpeculae	20 47.2	+27 52	4.44	+ 5	− 13	111.0	.43	17.6	.0018	969,180	+14.7
8571....	5.4	δ Cephei†	22 25.4	+57 54	5.37	+ 15	+ 3	85.4	.48	19.7	.0028	1,270,000	+12±
	7.0	RR Lyrae	19 22	+42 6	0.567	....	....	96.85	0.27	22.2	0.0006	166,500	−50.4

* All of the proper motions except for SZ Tauri, Y Ophiuchi, SU Cygni, and RR Lyrae are from the *P.G.C.* of Boss. The proper motion of SZ Tauri is from the *A.G. Berlin* catalogue, and that of Y Ophiuchi is deduced from a comparison of *Weisse's Bessel* and *A.G. Wien-Ottakring*.

† The visual magnitude of δ Cephei is 3.7.

0° and 180°. This condition seems to be independent of type, eccentricity, or region of sky. It seems reasonable to expect that small eccentricities may lead to comparatively large uncertainties in the angle ω, but it is doubted if that accounts for the few cases in Table II which differ from the majority.

TABLE II

H.R.	Star	*e*	ω
6616	Y Ophiuchi	0.16	202°
6863	Y Sagittarii	.16	32
7518	SU Cygni	.21	346
2650	ζ Geminorum	.22	333
	SZ Tauri	.24	77
	RR Lyrae	.27	97
6742	W Sagittarii	.32	70
7609	S Sagittae	.35	70
2332	RT Aurigae	.37	95
6616	X Sagittarii	.40	94
7988	T Vulpeculae	.43	111
8571	δ Cephei	.48	85
7570	η Aquilae	0.49	69

There is another possible explanation, however, viz., that some force, probably the gravitational attraction of the Milky Way, has tended to draw the axes of these orbits into its own plane. These stars are in two groups, three being in the region of the Milky Way not far from the solar antapex and the remaining ten in a group in the Milky Way near the apex. The mean residual velocity for the three stars near the antapex is -2.6 km and of the nine stars near the apex, $+2.5$ km.[1]

There appears to be some connection with magnitude, four stars from magnitude 3.5 to 4 (mean mag. 3.7) having an average radial motion of 5 km, while the remaining eight from magnitude 4.8 to 7.2 (mean mag. 5.8) give 14 km. These results are very consistent among themselves and lead to some confidence that the real velocities of the fainter stars of this class are greater than those of the brighter ones.[2] The possession of so many similar

[1] The abnormally high velocity of RR Lyrae was not used in these velocity results.

[2] Note added February 5, 1915. Subsequent investigation has shown that in the stars of class B also, the inherent velocity is a function of the magnitude.

characteristics indicates a close relationship of some kind and makes the Cepheid group an unusually interesting one.

Does this relationship extend back as far as their origin or is it simply selection, perhaps by the influence of the Milky Way, since they appear to be closely related to it? Can their unique variations in brightness be due to any of the peculiar characteristics of their orbits? It would seem that eccentricity combined with small orbital dimensions and small masses of the secondaries must be largely responsible for it.

The few cases of F- and G-type stars, other than the Cepheid and Geminid variables, with similar relative masses of the secondaries and orbital dimensions, have small eccentricities and show no variation in brightness.

The first suspicion which arises, especially as these stars are all approximately of the age of our sun, is whether they may not belong to a stream of which the sun is a member. This is negatived upon investigation by the fact that their radial velocities differ widely, not only in amount, but also in sign. The undoubtedly great distances of these stars indicate considerable masses for the primaries. On the other hand, the companions appear to be relatively small, conditions perhaps conducive to high eccentricity.

The considerable uniformity in relative masses of the secondaries, a probable similarity in masses of the primaries judging from their magnitudes, similarity in orbital dimensions and orbital velocities of the primaries, argue for unusual uniformity of physical and other conditions.

The following explanation has suggested itself to account for the variations of light of these systems. It is put forward, not as a finished theory, but in the hope that it may aid in the discovery of the complete one.

The actual variation of light is caused chiefly by changes in the light of the *secondary* due to disturbances in the part of its orbit near periastron, similar in principle to the brightening of a comet near perihelion. Where the angles of periastron of the primaries are about 90° as above, at the time of maximum velocity of *approach of the primary* (when the maximum of light occurs) both bodies

have passed periastron, the secondary receding from the observer. If we assume that there may be a lag in the production of the greatest amount of light after maximum gravitational disturbance (as seems almost certain to occur, judging from our experience with comets), the maximum of light might occur at *about* the time of maximum *approach of the primary.*

It should be observed that the coincidences are not exact (although the discordances are small), that the periods are short, and the discordances may be relatively appreciable. Similar reasoning leads to the occurrence of the minimum of light after the apastron passage at about the time of greatest velocity of recession.

It seems not improbable that the type of spectrum of the secondary at its maximum may be earlier, with few lines, thus causing the change of maximum intensity in the spectrum toward the violet observed by Albrecht and others. It may be pointed out that the velocity of the companion at periastron must be very high, in all probability sufficiently high to destroy any fine lines which might otherwise appear. Evidence bearing upon this matter is difficult to obtain with the high-dispersion spectrographs used for this work, on account of the long exposures required, perhaps also because the increase of brightness is not sufficient to bring out the characteristics of the spectrum of the secondary and because of the falling of absorption lines of one spectrum on bright regions of the other. It would seem that this matter could be best investigated with low dispersion on a bright star—where the exposures could be made very short with a view to detecting the spectrum of the secondary at maximum rather than the accurate measurement of velocities. It is also possible that a considerable increase of brightness may take place in the primary owing to the excitation of the near approach of the secondary.

Table III exhibits the intervals after periastron at which the maximum of light occurs and also the relation of minimum light to periastron. A glance at these results shows at once a relation in general between the length of period and the interval after periastron. Further examination shows also an apparent effect of eccentricity—the greater eccentricities hastening the maximum and the smaller eccentricities retarding it.

There is probably also an effect due to the relative masses of the secondaries, but on account of the uncertainties in this factor because of the unavoidable presence of the orbital inclination, it is difficult or impossible to allow for it.

TABLE III

Star	Maximum Light after Periastron		Minimum Light before Periastron
	Observed	Computed	
RR Lyrae	0^d06	0^d06	0^d05
SZ Tauri	0.34	0.32	1.23
RT Aurigae	0.26	0.45	0.95
T Vulpeculae	0.05	0.40	0.85
δ Cephei	0.59	0.47	0.87
Y Sagittarii	1.42	2.57	0.47
X Sagittarii	0.62	0.70	2.24
η Aquilae	1.06	0.75	1.09
W Sagittarii	1.64	1.91	1.02
S Sagittae	1.83	1.74	1.25
ζ Geminorum	6.38	2.67	0.18
Y Ophiuchi	8.45*	4.04	1.26*
SU Cygni	2.07	1.68	0.10

* The periastron position of Y Ophiuchi has been assumed to be in accordance with the light-curve, which seems to show more definite eccentricity than the velocity-curve.

I have attempted a general representation of the group upon the basis of the period, eccentricity, and mass-ratios, and find the best approximate representation to be from

$$I=+0^d27\cdot P\cdot\left(\frac{r}{r_1}\right)^2\cdot\sqrt{M},$$

in which

I = interval after periastron,
P = period in days,
r, r_1 = periastron and apastron distances, respectively,
M = mass-ratio $=\dfrac{m_1^3\sin^3 i}{(m+m_1)^2}$.

The computed intervals are given in Table III.

The representation appears to be better on the whole for those stars whose periastron angles are nearest to 90°, as shown by Table IV.

With the exceptions of ζ Geminorum and Y Ophiuchi, the representation appears to be as good as the present uncertainties justify us in expecting.

TABLE IV

	ω	O−C
Y Ophiuchi	22°?	+4ᵈ41
ζ Geminorum	333	+3.71
SU Cygni	346	+0.39
Y Sagittarii	32	−1.15
η Aquilae	69	+0.31
W Sagittarii	70	−0.27
S Sagittae	70	+0.09
SZ Tauri	77	+0.02
δ Cephei	85	+0.12
X Sagittarii	94	−0.08
RT Aurigae	95	−0.19
RR Lyrae	97	0.00
T Vulpeculae	111	−0.35

It may be noted that ζ Geminorum and Y Ophiuchi have the longest periods and have periastron angles of 333° and 202° (or 22°), differing considerably in this respect from the majority of the stars of the group. So far as such meager data may be interpreted, it seems to point to some sort of real difference of these two stars. SU Cygni, it may be noted, shows similar peculiarities. The star RR Lyrae, resembling the cluster variables, is included with the Cepheids, as it appears to have all of the characteristics of such stars.

In suggesting the foregoing explanation of the variations in brightness of the Cepheids I am not unmindful of the great weight which must attach to the relations between maximum and minimum light and maximum velocities of approach and recession first pointed out by Albrecht. These appear to be more than mere coincidences. The same appears to be true of the present explanation. It can hardly be mere coincidence, especially as it gives considerable evidence of resting upon well-known physical bases.

The two conditions must be harmonized in some way or explained, for as yet it is not clear why there should be any connection, except possibly in the cases in which the periastron is not far from 90°.

It is not at all clear why the periastron position should depend upon the production of maximum light in such a way as to bring the latter near the descending (or any) node of the primary as it appears to do in several of the foregoing cases.

If I may hazard a guess it would be that when (or if) enough data become available concerning these stars much light will be thrown, not only on the causes of their variations in brightness, but on the gravitational effects of the Milky Way itself.

Observatorio Nacional Argentino, Córdoba
February 20, 1915

AN APPARENT DEPENDENCE OF THE RADIAL VELOCITIES AND PROPER MOTIONS UPON MAGNITUDES AND SPECTRAL SUBDIVISIONS OF THE STARS WITHIN CLASS B

By C. D. PERRINE

During an investigation of the distribution of the nebulae and variable stars some peculiarities were suspected in the distribution and velocities of the stars of class B with reference to magnitude. The following tables exhibit the results of an examination of the stars contained in Campbell's catalogue in *L. O. Bulletin* No. 195.

The ranges of magnitude adopted are entirely arbitrary for the sake of convenience.

TABLE I

Limits of Magnitude	Galactic			Non-galactic			All			
	No. of Stars	Residual	Mean V_1	No. of Stars	Residual	Mean V_1	No. of Stars	Residual	Mean V_1	Percentage
		km	km		km	km		km	km	
2.2 and brighter..	15	+0.6	3.2	3	−0.5	2.7	17	+0.4	3.2	0
2.3–2.9..........	14	+3.5	5.9	2	−5.1	9.9	13	+2.3	6.7	23
3.0–3.9..........	38	+1.1	5.6	5	−1.9	2.1	43	+0.8	5.2	14
4.0–4.9..........	80	−1.0	6.2	33	−2.4	6.7	112	−1.6	6.6	19
5.0..............	21	−2.5	9.3	15	−2.0	6.4	36	−2.4	8.2	31
Variables........	......	......	......	......	......	......	4	−1.6	5.5	0
*4.0–4.9........	......	......	......	12	−2.1	9.0	......	......	......	......

* Galactic latitudes ±40° to ±90°.

Note.—The last column contains the percentages of stars in each group which have velocities of 10 km and over.

The galactic limits were taken as 20° either side of the mean galactic plane, embracing an area a little more than one-third of the entire sky.

An examination of the individual results shows that if we separate the first group into two—stars of 1.5 and brighter, and stars fainter than 1.5—we have the following:

	No. of Stars	Residual V	Mean V
		km	km
1.5 and brighter.............	4	+0.3	0.5
1.6–2.2.....................	13	+0.3	4.1

The group 2.3–2.9 appears to be abnormal, owing to the effect of three large values of V, in the small number of 13. If we reject these three, the value of residual V becomes +1.0 and of mean V, 3.8, which agree well with the others.

As the B8 and B9 stars appear, in some respects at least, to be somewhat different from the other stars of class B, the effect of omitting these stars on the foregoing investigation was tried. The result is given in Table II for all stars.

TABLE II

	No. of Stars	Residual V	Mean V
		km	km
2.2 and brighter	15	+0.3	3.4
2.3–2.9	12	+1.3	6.1
3.0–3.9	38	+0.3	5.5
4.0–4.9	88	−0.7	6.2
5.0	24	−2.7	8.9

In Table III are given similar data for the B8 and B9 stars.

TABLE III

	No. of Stars	Residual V	Mean V
		km	km
2.2 and brighter	2	− 1.6	1.6
2.3–2.9	1	−14.5	14.5
3.0–3.9	5	− 4.2	4.7
4.0–4.9	24	− 4.6	7.7
5.0	12	− 2.0	7.0
All	45	− 3.9	7.0
Galactic	23	− 2.5	4.3
Non-galactic	22	− 5.3	9.8

An examination of the individual velocities shows considerable uniformity within the groups; the wide differences between the extreme groups is evident at a glance.

An investigation of the proper motions of these stars also reveals an apparent dependence upon magnitude as in the following tables.

TABLE IV

CLASS B–B5 ALL PARTS OF SKY

Magnitude	No. of Stars	Mean μ
2.2 and brighter.....	16	0″047
2.3–2.9.............	12	.032
3.0–3.9.............	38	.028
4.0–4.9.............	88	.024
5.0.................	24	0.025

TABLE V

B8 AND B9 STARS ALL PARTS OF SKY

Magnitude	No. of Stars	Mean μ
2.9 and brighter.....	4	0″088
3.0–3.9.............	5	.049
4.0–4.9.............	24	.046
5.0.................	12	0.040

TABLE VI

B–B5

	GALACTIC		NON-GALACTIC	
	No. of Stars	Mean μ	No. of Stars	Mean μ
2.2 and brighter..........	12	0″027	3	0″087
2.3–2.9..................	12	.033	0	....
3.0–3.9..................	35	.029	3	.021
4.0–4.9..................	66	.024	5	.013
5.0......................	18	0.024	4	0.030
All......................	143	0.026	15	0.034

The non-galactic results in these investigations have but little weight owing to the few stars of this class which are situated far from the galaxy. It was suspected that the proper motions of the non-galactic stars might be greater than those of the galactic stars, brightness for brightness, and that the fainter stars, being more widely distributed, might have caused the apparent dependence upon magnitude when in reality it was a dependence solely upon

galactic latitude. This point will be investigated further in other classes of stars where the conditions are more favorable.

TABLE VII

B8–B9 STARS

	GALACTIC		NON-GALACTIC	
	No. of Stars	Mean μ	No. of Stars	Mean μ
2.9 and brighter..........	3	0."063	1	0."162
3.0–3.9..................	4	.047	1	.058
4.0–4.9..................	14	.032	10	.070
5.0......................	4	.040	8	.040
All......................	25	0.039	20	0.062

Attention should be called especially to the apparent contradiction between the radial velocities and proper motions so far as dependence upon magnitude is concerned—the velocities appear to *increase* with decreasing brightness whereas the proper motions *decrease*.

In view of the peculiar systematic error which appears to exist in the radial velocities of practically all stars, and its particularly large value for the class B stars, no attempt is made at present to analyze this matter further.

The proper motions have not been cleared of the effects of the motion of the solar system, which is somewhat uncertain owing to the necessity of making assumptions as to distances. Any such effects with respect to magnitudes are probably practically negligible. There may be a noticeable effect between the galactic and non-galactic results, but it is believed hardly enough to wipe out the entire differences shown above.

The spectral subdivisions in stars of class B appear to show a similar increase in proper motion as found by Professor Boss for class A stars.

It was suspected that the real cause of this variation might be spectral subdivision rather than simply magnitude. There is an undoubted tendency for the brighter stars to belong to the earlier subdivisions, but the dependency seems to be chiefly upon magnitude, as shown by Table VIII.

TABLE VIII

Class	2.9 and Brighter		3.0 and Fainter	
	No. of Stars	Mean V_1	No. of Stars	Mean V_1
		km		km
B -B2	19	3.7	34	7.1
B3-B5	8	6.7	113	6.3
B3-B5*	6	3.2	....	....
B8-B9	3	5.9	41	7.1
All	30	4.7	188	6.6

* Rejecting two large values of V.

The effect was tried on the B–B5 stars using the values V_2 instead of V with the following results:

TABLE IX

	No. of Stars	Residual V_2	Mean V_2
		km	km
2.2 and brighter	15	+1.5	3.8
2.3–2.9	12	+5.0	6.2
3.0–3.9	38	+2.2	5.5
4.0–4.9	88	0.0	6.5
5.0	24	−1.3	8.4

From these data the following conclusions are tentatively drawn:

A. That the average radial velocities of the B stars discussed depend in general upon magnitude, the *velocity increasing* with *decrease of brightness.*

This seems to be true for galactic and non-galactic regions and to some extent true also for the stars of classes B8 and B9. In these latter stars there appears to be a great difference between the velocities in the galactic and non-galactic regions.

B. There appears to be a relation in general between the brightness of these stars and their type—the brighter stars "preferring" the early classes—B, B1, and B2, the fainter stars B3, B5, B8–B9.

C. The *residual* radial velocities (V_1) of the brighter B stars within 20° of the plane of the Milky Way show a progressive tendency with magnitude—i.e., a small *positive* radial velocity for

the group 2.2 magnitude and brighter which changes to a *negative* radial velocity for the stars fainter than 4.0 magnitude.

D. The residual velocities of the non-galactic B stars appear to be consistently *negative* for all magnitudes. Of the 58 non-galactic stars discussed, 22 have positive values and 36 negative values.

E. The proper motions of the B stars appear to be functions of the magnitudes.

The B–B5 stars have been charted in groups by magnitude, also the B8–B9 stars and those with velocities of 10 km and over.

The following conditions regarding their distribution are indicated:

M. All of the charts show that the plane of distribution of the B stars does not coincide exactly with the plane of the Milky Way but falls *below* or *southwest* of it in the regions $\alpha 3^h$–$7^h \pm$ and *above* or *north* in the region 12^h–16^h.

It may be noted that the first region is that of the Orion Nebula and the second that of the Eta Argus-Crux region of the Milky Way. These tendencies appear to strengthen the connection of these stars not only with the galaxy but with the most important nebulous regions.

N. The apparent distribution of these B stars is a function in general of the magnitude—the fainter stars being found over a wider area than the brighter ones.

An examination of the magnitudes of the B8 and B9 stars shows that in general they are faint, only eight being brighter than fourth magnitude and three brighter than the third magnitude, out of 45 examined. In the matter of distribution these B8 and B9 stars appear to be more widely scattered than the B–B5 stars of equal magnitudes, as pointed out by Campbell.[1]

Two additional tendencies are to be noticed:

A number of the fainter stars, including B8 and B9 and stars of large velocity, are found in the region not far from the two Nubeculae.

The region of the Milky Way and sky from 18^h to 0^h is entirely free from the brighter stars and almost entirely so from the stars

[1] *Lick Observatory Bulletin No. 195*, p. 107.

fainter than 5.0, *but contains the normal proportion of stars 4.0–4.9.*

There appears to be some tendency to grouping of stars with similar velocities. These have not been studied in detail.

A consideration of related facts, including proper motion, and having in mind the large difference in mean radial velocities between the B and A stars, indicates that the large range found above in the mean velocities within the B stars is real.

Should these conditions prove to be representative of the group, as appears entirely probable, it would seem that here we have further evidence of being near the origin of these stars—and strong evidence that the velocities of translation of the matter forming these stars was low during their early stages of development.

CONCLUSIONS

The following conclusions may be summarized from this investigation:

1. Within class B the inherent velocity of the stars is in general a function of the brightness and of the spectral subdivisions, these two conditions being related.

2. The radial velocities at the beginning of the series, i.e., the brighter stars and those of the early spectral subdivisions, are very nearly zero.

3. In general the fainter stars and those of later subdivisions have inherent velocities of between 8 and 9 km per second—connecting well with the values of 8.6 km for Ap and 10.3 km for A found in an investigation of the class A stars.

4. The apparent distribution of the B stars in the sky with respect to the Milky Way is also a function of magnitude, spectral subdivision, and inherent radial velocity, the fainter, later spectral subdivisions and larger inherent velocities being found farther from the galactic plane.

5. A variation of residual radial velocity for each group according to magnitude, etc., is indicated, but in view of the probable rapid variation of internal conditions it seems not advisable to draw any conclusions until more data are at hand. If this progressive effect is sustained such a condition is very suggestive.

The foregoing results may be considered to strengthen further the discovery by Campbell, Kapteyn, Frost and Adams of the variation of inherent velocity of the stars with supposed increasing age. It further tends to place the early velocity close to zero and to show that the velocities increase regularly and not by jumps.

Observatorio Nacional Argentino
Córdoba
February 8, 1915

Minor Contributions and Notes

SOME RECENT DISCOVERIES IN SPECTRUM SERIES

During the past few years our knowledge of the subject of spectrum series has been very greatly enlarged. The classical work of Kayser and Runge left many physicists with the impression that little more was to be done, but the "combination principle" of Ritz served as a starting-point for a great advance. The larger part of this advance we owe to Paschen and to Fowler. During the past year a Doctor's dissertation (Tübingen) by E. Lorenser, working under Paschen, has furnished some new results, and a remarkable contribution to the subject has just been made by Fowler in his Bakerian Lecture (*Phil. Trans.*, **214**, 225, 1914). The present brief review of these two pieces of work is presented in the hope of making plain to those interested in atomic vibrations the nature of the problem which awaits the man who seeks to devise a working model of an atom.

The work of Lorenser includes some excellent measurements, chiefly in the red, on the spectra of Mg, Ca, Sr, and Ba, and a presentation of some of the series of these elements, of which a few are new, and some perhaps a little doubtful. Fowler's paper deals largely with series in the "enhanced" spectrum of Mg, including a survey of some of the series of other elements also. As the mass of material in these two papers is too great for condensation into a brief review, it seems sufficient to consider Mg alone, since we know rather more about its spectrum than about those of kindred elements, and because it is an interesting and probably typical system.

A brief introductory statement about notation will help. Every series consists of a converging arrangement of lines, or "members," each of which may be complex and contain from one to six lines. The difference in "wave-number" (number of waves

per cm) between the limit of the series and each member is called the "term." A series may be approximately represented by a formula, the exact nature of which is not known, and is not always important; one type (Mogendorff and Hicks) is

$$\nu = \frac{1}{\lambda} = \text{limit} - \frac{N}{\left(m+a+\frac{b}{m}\right)^2}$$

in which N is the "universal series constant," a and b are constants of the individual series, and m is a variable integer. This formula may be symbolically written $\nu = \text{limit} - (m,\ a)$ which has degenerated in practice to $\nu = \text{limit} - ma$. The limit itself is commonly a "term" of some other series, and the different series are distinguished by appropriate letters for the constants, p, s, d, and f, so that the abbreviated designation of the formula becomes $1s - mp$, or $2d - mf$, or whatever the case may be.

The theory recently published by Bohr indicates that in certain cases the constant N may have to be replaced by $4N$, and it is possible that N may apply to those series whose vibrations are due to the neutralization of an atom lacking one electron, while $4N$ is required when the atom, having lost two electrons, regains one and vibrates as it does so.

The spectrum of Mg consists of three main systems, composed of triplet, pair, and single-line series. Of these, the pair system has been greatly extended by Fowler's latest work, and he has shown that the constant $4N$ is required for all the series of this system, while N alone is proper for all other series in Mg. The same remark applies to corresponding series in kindred elements, and also to what was supposed to be the principal series of hydrogen, which Bohr and also Fowler now regard as due to helium. This whole group of series is variously designated as "spark" or "enhanced" series, because of their behavior, though these designations are not ideal, as the lines may be obtained from many different sources of light.

Within each of these three systems there are, in general, four different *types* of series, known as "principal," "sharp" (second subordinate), "diffuse" (first subordinate), and "fundamental"

(Bergmann). These names are not always very appropriate; for instance, the "sharp" series is often diffuse, and the "fundamental" series may be composed of the faintest lines in the spectrum; but this is not vital. Each type gives a series of "terms" which are, of course, quite different from one another. They may be listed as mp, ms, md, and mf for the triplet system; $m\pi$, $m\sigma$,

TABLE I

SERIES IN THE SPECTRUM OF MAGNESIUM

Formula	Series Letter	Remarks	Fowler's Designation
Pair system—			
$1\sigma - m\pi$	π	Principal series, wide pairs	P
$1\pi - m\sigma$	σ	Sharp series, wide pairs	S
$1\pi - m\delta$	δ	Diffuse series, wide pairs	D
$2\delta - m\phi$	ϕ	Fundamental series, narrow pairs	4481 series
$3\delta - m\phi$	ϕ'	Very narrow, observed as single	A
$3\phi - m\phi$	ϕ''	A single-line series in the pair system	B
$2\sigma - m\pi$	π'	Minor series of principal type	$FP(p)$
$2\delta - m\pi$	π''	Same. One member observed	C
$2\pi - m\sigma$	σ'	Minor series, sharp type	$FP(s)$
$2\pi - m\delta$	δ'	Minor series, diffuse type	$FP(d)$
Triplet system			
$1s - mp$	p	Principal series, wide triplets	
$1p - ms$	s	Sharp series, wide triplets	
$1p - md$	d	Diffuse series, wide triplets	
$2d - mf$	f	Fundamental; narrow; observed as single lines	
$1p - mf$	f'	(Possible) series of narrow triplets	
$1p - mp$	p'	(Possible) series of principal type	
Single-line system—			
$1S - mP$	P	Principal series, runs into Schumann region	
$1P - mS$	S	Sharp series (Fowler)	
$1P - mD$	D	Diffuse series, Rydberg's	
$2S - mP$	P'	Minor series of principal type	
$1P - mP$	P''	Another of principal type	
Inter-system series—			
$1S - mp$	Sp	Single-line series with principal-type terms	
$2D - mf$	Df	(Possible) series of fundamental type	

$m\delta$, and $m\phi$ for the pair system; and mP, mS, mD, mF for the single-line system. These series of terms recur in different parts of the spectrum, converging to different limits, thus producing "shifted" or "parallel" series, two of which are exactly alike in all wave-numbers, if a constant is added to the wave-numbers of one of them. Series of the principal type consist (unless they are

single lines) of groups of lines which converge to a single line as a limit: series of the sharp and fundamental types are made up of groups whose wave-number differences are constant; while series of the diffuse type combine both sets of features, the group constituting the series-member being complex at first, but converging toward a simple group of the same character as the corresponding sharp series.

The Rydberg-Schuster law was the forerunner of others of the same sort. All of them can be grouped into the statement that the limit of a series is itself a term of some other series. In some cases, however, the first "term" of a series must be formed by treating the wave-number as negative, and in these cases the corresponding series-member occurs in the spectrum wrong way around, that is, with the line which is expected to have the shorter wave-length appearing on the long-wave side, and the intensities also reversed.

With this preamble, then, the table of series of Mg here given may take on some significance. In our abbreviated notation some differences exist in practice as to the numeration adopted for m; the values here used correspond to Fowler's latest work and differ from those of Ritz, Paschen, or Lorenser. It must be noted that each term $m\pi$ has two values (since these series-members are double) which may be designated separately, if desired, as $m\pi_1$ and $m\pi_2$; δ is also double, and p and d are triple. The second column of the table contains a suggested system of series lettering which may be convenient for reference. In the single-line system it will be noticed that no series of the fundamental type has yet been found. Such series exist, however, in certain other elements. At the bottom of the table there is a section devoted to "inter-system" series. These are derived in a way sufficiently indicated by their formulae, and they are extremely interesting, as they are combinations between the triplet and single-line systems. It is significant that no combinations are known between the pair system and either of the others; this fact, together with the use of $4N$ in the formula, indicates that the pairs originate in a different vibrating atom from the other lines; but the single lines and the triplets are apparently given by the same kind of atom. The inter-system series are relatively more conspicuous in other spectra (e.g., Hg).

There is little doubt that still more series are to be discovered in the Mg spectrum, but, as it stands, the entire system involves twenty positively established series and some other likely ones. This is a sufficiently formidable complex to alarm the bravest constructive mathematical physicist, but we may be thankful that it is no worse. The series are quite closely interrelated, most of the known lines in the spectrum are now properly labeled, and we have a handy filing system adapted to the reception of new series as they are discovered.

A word should be added in regard to the sources of light required to bring out these various series. All the systems are represented in the spectrum of the ordinary arc in air (as well as the vacuum "vapor lamp"). The triplet and single-line systems are well developed in the arc, the latter best near the positive terminal. The single-line system is strongly given by the electric oven, as is also the series called *Sp* in the table. The vacuum arc emphasizes the pair system, especially in the region close to the negative terminal, from which Fowler obtained many of his new series; the stronger lines of this system occur also in the spark in air, but are too diffuse for accurate observation. For the outer members of most series the best conditions seem to be a discharge through a considerable depth of luminous vapor at a low pressure.

F. A. Saunders

Vassar College

Reviews

Polhöhen-Schwankungen. (Sammlung Vieweg, Heft 11.) By E. PRZYBYLLOK. Braunschweig: Friedr. Vieweg & Sohn, 1914. 8vo, pp. 41, figs. 8. M. 1.60.

The "Sammlung Vieweg" has set itself the task, so its advertisement reads, of making more widely known through clearly written, compact discussions those realms of research and those scientific theories that are at the present time in a state of development; and, in particular, it undertakes to show the present status of the various problems. Through this procedure it hopes to facilitate the ready comprehension of modern scientific developments and to point out the direction in which further research is to proceed.

This ambitious series of small treatises, which already includes memoirs on flying machines, gravitation, and sugar beets, is capably represented astronomically in the booklet that is the subject of the present review. Dr. Przybyllok does for the problem of latitude variation exactly what the editors have advertised. He presents his subject briefly, clearly, thoroughly, and without introducing details of the serious mathematical discussions that are at the basis of so many phases of the practical investigation. The history of the sixty years of suspicion that culminated finally in the general admission of the inconstancy of terrestrial latitude, the development of the problem during the last thirty years into an international project, the theories and suggestions that have been advanced to account for the observed and suspected motions of the pole and for the ever-present systematic deviations at different stations, and finally the present state of our knowledge of latitude variation—all are treated here in sufficient detail for the purpose, and the most significant literature of the subject is cited throughout in the footnotes. Charts showing the path described by the pole from 1890 to 1913 are given on a folded insert at the back of the book.

As might be expected, much of the latter half of the treatise deals with the detection and measurement of the elusive Kimura term and with the interpretative speculations concerning it. Some of the latest considerations on this point, however, are not mentioned. None of the two or three small errors noticed throughout the work are worth mentioning,

unless one would care to remark that the Hawaiian Islands are in the Northern Hemisphere.

It is interesting in this time of serious international political disturbances to observe that the development of the study of latitude variations has depended entirely on peaceful international alliances. Nearly all of the great governments have taken part in the latitude service, co-operative observations have been made toward the solution of this one problem in all parts of the world, and men of a dozen different nationalities have worked together toward the same goal. What effect the present world-conflicts may have on the continuity of the international latitude investigations is uncertain. The possibility that the work in the United States may suffer is suggested in the resolutions recently adopted by the American Astronomical Society and by the Astronomical Society of the Pacific asking the government to make some special provision for the maintenance of the stations in this country.

HARLOW SHAPLEY

Données numériques de spectroscopie. Extrait du Volume III des *Tables annuelles de constantes et données numérique.* Paris: Gauthier-Villars et Cie, 1914. (Chicago: The University of Chicago Press.)

Portions of Volume III of the "International Tables of Constants and Numerical Data, Chemical, Physical, and Technological" have been issued in separate folios as follows:

Title of Folios	Editor	No. of Pages	Price in Francs
A. Spectroscopie	L. Brüninghaus	74	10
B. Électricité, Magnétisme Conductibilité des électrolytes Forces électromotrices	Dr. L. Mahlke Professor W. C. McG. Lewis Professor F. Dutoit	79	10
C. Électronique, Ionisation, Radioactivité	J. Saphores	9	2.50
D. Cristallographie et Minéralogie	L. J. Spencer	19	4
E. Biologie	L. Terroine	17	4
F. Art de l'Ingénieur Métallurgie	G. Fiek, W. Hinrichsen S. L. Archbutt, Portevin, Nusbaumer	74	10

The folio on spectroscopy, which should be of especial interest to readers of the *Astrophysical Journal,* comprises pp. 165–238 of the complete volume. It contains an introduction by M. H. Deslandres, a list

of chapters, and a list of the substances for which spectroscopic data are given. It is grouped under the following chapters: (i) "Description of Emission Spectra"; (ii) "Zeeman Effect"; (iii) "Displacement or Broadening of Spectral Lines under Pressure"; (iv) "Relations between the Intensity of Spectral Lines and the Conditions of Excitation"; (v) "Velocity of Propagation of Luminous Vapors in the Spark"; (vi) "Series Grouping of Lines or Emission Bands or Absorption Bands"; (vii) "Absorption Spectra of Gases and Vapors of Elements and Inorganic Substances"; (viii) "Absorption Spectra of Gases and Vapors of Organic Substances"; (ix) Absorption Spectra of Inorganic Substances in the Solid or Liquid Form or in Solution;" (x) "Absorption Spectra of Organic Substances in the Solid or Liquid Form or in Solution"; (xi) "Absorption Spectra of Various Substances of Animal or Vegetable Origin."

Under each chapter head the titles are arranged in alphabetical order of the chemical symbols or in alphabetical order of the names of the bodies. There are, in all, 146 tables of data. It is a considerable advantage to have these separate folios available, as the complete volume is not only too expensive for very wide circulation but is rather cumbersome.

H. G. G.

The Earth, Its Life and Death. By A. BERGET. Translated by E. W. BARLOW. New York: Putnam, 1915. Pp. 366. $1.50.

With the entertaining vivacity of style for which the French are celebrated, the author presents to the well-educated layman some of the more interesting facts known about the earth as a planet. The central chapters deal with the form and mass of the earth, its motions as a planet, volcanic and seismic phenomena, gravity, electricity, and magnetism of the earth, the circulation of the winds and oceans, and other physical considerations. In the opening chapter the origin of the earth is sketched, largely according to the time-honored theory of Laplace. The nebula is imagined to have been one of the spiral type and is explained by the actual collision of two dark suns. An appropriate old age and death of the earth and solar system are supposed to be conditioned by the progressive radiation of heat which it is thought will render the earth uninhabitable within a few million years.

Here and there the volume contains evidence that the author is not fully abreast of modern progress in geology and astronomy, although his familiarity with physics seems to be of a much higher order. Some

of the lapses have been tactfully corrected by the translator but these are mostly cases of detail. In the reviewer's estimation, the greatest disadvantage of the book is closely related to its charming style. Many ideas which are really highly speculative are so ingeniously woven in with well-determined facts or established theories that the argument appears most plausible. It is doubtful whether the lay reader will be sufficiently on his guard, or will have the necessary technical knowledge, to enable him to discriminate between what is reliable in the book and what is strongly colored by the author's somewhat naïve deductions.

E. B.

Astronomy. By G. F. CHAMBERS. New York: D. Van Nostrand Co., 1913. Pp. xxiv+335, figs. in text 68, plates 135. $1.50.

In this profusely illustrated volume, with text that is in general simple and straightforward, there is much that is very attractive. But, on the other hand, there is revealed a narrowness and an inappreciation of modern advances, that makes the work savor of the pre-spectroscopic age. True, in the introduction, the author states, "It will be my endeavor to keep as closely as I can to astronomy in the older and more limited sense." No fault need be found with this limitation providing the author does not at the same time close his mind to the testimony from astrophysical sources bearing on the topics considered. Fields are left barren, phenomena are left unexplained, or suggested explanations, for which the older astronomy furnishes no test, are left vague, though results, sound explanation, and proof are available from other sources. A few samples will serve to illustrate:

On p. 214, concerning the Algol variables, "By way of explanation it has been suggested that a non-luminous satellite revolves round the primary star and eclipses it at stated intervals." A mere suggestion, nothing more. On p. 232 the author professes himself as unconvinced of the gaseous nature of the so-called gaseous nebulae. The observers of the astrophysical observatory of the Smithsonian Institution will be grieved to read on p. 23, "Though attempts have been made to calculate and state by figures the heat-giving power of the sun, it must be obvious that all such calculations can only be, if not quite imaginary, yet very wild and indeterminate." On p. 309, "Though the whole idea of such an attempt to prove stellar movement by means of the spectroscope seems a high flight of the imagination, yet it does not appear that there are sufficient grounds for distrusting the results arrived at in the case of several dozen stars, though these results require us to talk about

miles per second as the pace at which the stars in question are travelling to and from somewhere." On p. 122, and again on p. 306, it is stated that helium is not found on the earth.

But even in the field to which this book is ostensibly devoted it is not free from fault. There is no mention of the stellar parallax problem, and proper motion is dismissed in a phrase. On p. 242 he states, "The Milky Way . . . . is one vast nebula running around the heavens in the form of a belt." When we think of the great nebulous fields in Ophiuchus and Sagittarius, this seems not so fantastic did we not find on p. 241, "Subject to certain special exceptions, the fixed stars are distributed fairly evenly over the whole sky." These special exceptions do not seem to refer to the gradual thinning out of stars with increasing galactic latitude. On p. 83 we find the term "germination," in quotation marks, applied to the doubling of Martian canals. On p. 112 the asteroid (699) is noted as the one having the most and least eccentric orbit. On p. 110 it is stated that the only asteroids worthy of consideration are the four first discovered and Eros. Even the Achilles group is apparently of no interest. A considerable discussion of the solar rotation makes no mention of the most interesting thing about it, the equatorial acceleration. On p. 8, "Spots often start from a pore as a place of origin." This is most decidedly not the case. It is interesting to contrast the degree of positiveness of the following two statements: concerning sun-spots, p. 15, "The periodicity of 11.1 years is now established to a *dead certainty*"; and concerning Neptune, p. 106, "The existence of one satellite *seems* to be assured." On p. 228, in speaking of the nebulous matter in the Pleiades, the astonishing statement is made that it has had its origin in the last half-century. Similar evidence might convince one that stars below the naked-eye limit were created about 1610. Just one other matter, and that is concerning the vivid and very misleading representation of the colors of double stars in plates 84 and 85. It is said that if one inverts himself by standing on his head, color contrasts are greatly enhanced. I doubt if less drastic revolution of method of observing would reveal the colors as represented.

There is perhaps little of commendation in what precedes; there is certainly no notion to recommend the book as a reference authority. Nevertheless in it there is much that is admirable and many readers will doubtless get a taste for astronomy from it. But, on the whole, the author cannot be said to have enhanced the esteem with which he was regarded as the author of Chambers' *Handbook of Astronomy*.

P. F.